Exercise List 3
Market Failure: Markets with Adverse Selection

Exercise 1. A good of two qualities, high (H) and low (L), is traded in competitive markets in which each seller has a single unit and each buyer wants to buy a single unit. There are n_H sellers with a unit of high quality whose opportunity cost is c_H euros, n_L sellers with a unit of low quality whose opportunity cost is c_L euros, and n buyers who value a unit of high quality in u_H euros and a unit of low quality in u_L euros. Assume that $u_H > c_H > u_L > c_L$.

(a) Suppose that quality is observable. Calculate the competitive prices for the cases $n > n^H + n^L$ and $n < n_H + n_L$. Discuss if these competitive equilibria generate the maximum surplus. (If you find it helpful, assume $u_H = 10$, $c_H = 7$, $u_L = 5$, $c_L = 0$, $n_H = 1$, $n_L = 1$ and $n \in \{1, 3\}$.)

(b) Now suppose that quality is not observable and both qualities trade in the same market. Also assume that $n = n_H + n_L$. Represent the supply and demand schedules in the plane (q, p) and calculate the competitive equilibria of this market when the expected value of a random unit,

$$u(n^H, n^L) = \frac{n_H}{n_H + n_L} u_H + \frac{n_L}{n_H + n_L} u_L,$$

is greater than c^H, and when it is less than c^H. (If you find it helpful, use the parameter values suggested in part (a), and consider the cases $n_L = 1$ and $n_L = 2$.)

Exercise 2. Consider a market for used cars whose qualities, indexed by the sellers' cost, are uniformly distributed in the interval $[2, 6]$. Buyers are risk-neutral and value each quality 20% more than sellers. Naturally, each seller knows the quality of the good he sells, but quality is not observable to buyers prior to purchase. Assume that there are more buyers than sellers.

(a) Determine the market supply and the average quality of the cars offered at each price.

(b) Calculate the market equilibrium.

Exercise 3. Consider an insurance market in which all individuals have the same initial wealth $W = 1$ and the same preferences, which are represented by the von Neumann-Morgenstern utility function $u(x) = \sqrt{x}$, where x is the individual's disposable income. Each individual faces the risk of having an accident resulting in losing his wealth. For a fraction $\lambda \in (0, 1)$ of individuals the probability of having

this accident is $p^L = 1/2$ whereas for the remaining fraction $1 - \lambda$ this probability is $p^H = 4/5$. Insurance companies know this information, but at the time of signing a policy do not observe whether the probability of having an accident for a particular individual is p^L or p^H.

(a) If it is mandatory that policies offer full insurance, which policies will companies offer for each value of λ? Which individuals would subscribe them?

(b) If companies are free to offer any policy, which policies will be offered for each value of λ? (Here you need to identify a separating equilibrium.)