Exercise List 2
Market Failure: Market Power, Externalities and Public Goods

Exercise 1. Consider an economy there are two goods, time (l), which is used for labor and leisure activities, and consumption (c). There is a *worker* who has no resources other than her labor income, and her preferences over consumption and labor (the counterpart of leisure) are represented by a utility function $u(l, c) = -l^2/2 + c$. The economy has no endowment of consumption, but there is a technology that allows to produce consumption using labor l as input according to the production function $f(l) = 2\sqrt{l}$. There are also $n = 2$ *owners* each of whom manages a firm that produces consumption with the existing technology. Owners supply no labor and only care about the consumption they can procure using the profits of the firms they manage.
(a) Calculate the economy's competitive equilibrium. (That is, the equilibrium that will result if the worker and onwers act as price-takers.)
(b) Calculate the economy's oligopolistic equilibrium. (That is, the equilibrium that will result if the worker acts as a price-taker, but the owners behave strategically, acting as oligopolists in the market for consumption, and as oligopsonists in the labor market.)
(c) Determine the impact of a minimum (real) wage $\bar{\omega} = 1$ in the economy's equilibria identified in (a) and (b).

Exercise 2. There are two goods, x and y, and two persons, A and B, with preferences described by the utility functions $u_A(x_A, x_B, y_A) = 2x_A + \alpha \ln x_B + y_A$, where $\alpha > 0$, and $u_B(x_B, y_B) = x_B + y_B$, respectively. Note that Person A cares directly about how much Person B receives of the x-good. No production is possible, and the economy's endowments are $\bar{x} > \alpha$ and $\bar{y} > 0$. Determine all of the Pareto optimal allocations in which each person receives a positive amount of each good.

Exercise 3. The de Beers Brewery uses water from the Pristine River in its brewing operations. Recently, the United Chemical Company (also called Chemco) has opened a factory upstream from de Beers. Chemco's manufacturing operations pollute the river water: let x denote the number of gallons of pollutant that Chemco dumps into the river each day. De Beers's profits are reduced by x^2 dollars per day, because that's how much it costs de Beers to clean the pollutants from the water it uses. Chemco's profit-maximizing level of operation involves daily dumping of 30 gallons of pollutant into the river. Altering its operations to dump less pollutant reduces Chemco's profit: specifically, Chemco's daily profit is reduced by the amount $(30 - x)^2/2$ if it dumps

$x \in [0, 30]$ gallons of pollutant per day. There are no laws restricting the amount that Chemco may pollute the water, and no laws requiring that Chemco compensate de Beers for the costs imposed by Chemco's pollution.

(a) Coase's argument holds that the two firms will reach a bargain in which a Pareto efficient level of pollution will be dumped. Determine the efficient level of pollution. If efficiency requires that $x < 30$, then determine the range of bargains the two firms could be expected to reach– i.e., the maximum and minimum dollar amounts that de Beers could be expected to pay to Chemco in return for Chemco's agreeing to dump only x (less than 30) gallons per day.

(b) Now suppose a law is passed that requires anyone who pollutes the Pristine River to fully compensate any downstream firm for damages caused by the polluter's actions. How does this change the Pareto efficient level of pollution? How does it change the pollution level that Chemco and de Beers will agree to? How does it change the payments that one of the firms will make to the other?

(c) Now suppose that de Beers is not the only firm harmed by Chemco's pollution: There are two more brewing firms whose profits are reduced by the pollution in the same amount as de Beers. How would this affect your answers in (a) and (b)?

Exercise 4. Each resident of the tiny country of DeSoto owns a car and has a positive endowment of *simoleans*, the economy's only tangible commodity. Cars use simoleans for fuel: every mile that a car is driven uses up c simoleans, and puts $b > 0$ units of hydrocarbons into the air. All residents enjoy driving and consuming simoleans, but dislike air pollution. A resident's preferences are represented by a utility function of the form $u_i(x, y, H) = v_i(x) + y - \alpha_i H$, where x and y denote the resident's miles driven and consumption of simoleans, respectively, $H = b(x_1 + ... + x_n)$ is the level of hydrocarbons in the air, and v_i is a strictly concave and increasing function. Consider only the allocations in which each x_i and each y_i is positive.

(a) Give the n marginal conditions that characterize' the Pareto optimal allocations, and interpret them in words.

(b) Give the n marginal conditions that characterize the competitive equilibrium, and interpret them in words.

(c) Determine whether, in the equilibrium, all families necessarily drive "too much," all families necessarily drive "too little," or whether the miles driven might be either too large or too small, depending upon the data of the problem.

Exercise 5. A certain restaurant is known for refusing to give separate checks to customers: After a group has ordered and eaten together at this restaurant, the group is presented with a single check for the entire amount the group has eaten. It has been suggested that the restaurant does this because with a single check those who dine in groups will be likely to simply divide the charge equally, each person paying the same amount irrespective of who ordered what; and that diners, knowing that they will ultimately divide the charge equally, will order more than they would have ordered had each paid for his own order. Analyze this suggestion in the following setting: There are n diners in a group. Each has a utility function of the form $u(x_i, y_i) = y_i + \ln x_i$, where x_i is the amount of food that i ordered and y_i is the amount of money that i has after leaving the restaurant. The restaurant charges p dollars per unit of food, and the restaurant's profit is an increasing function of the amount of food it sells at the price p. Each diner knows when he orders his food that the group will divide the check equally. Compare the outcome under this arrangement with that resulting when each diner pays for his own order. What are the restaurant's profit and the diners' welfare in each case?

Exercise 6. Alpha and Beta have just terminated their marriage. They hold no animosity toward one another, and each is concerned about the welfare of their only child, little Joey. Their preferences are described by the utility functions $u_A(x, y_A) = x^\alpha y_A$ and $u_B(x, y_B) = x^\beta y_B$, where y_A and y_B denote the number of dollars "consumed" directly by the respective parents in a year, and x denotes the number of dollars per year consumed by Joey. Joey's consumption is simply the sum of the support contributions by Alpha and Beta, $s_A + s_B$. These contributions will be voluntary: Neither parent has sought a legal judgment against the other. Assume throughout that $\alpha = 1/4$ and $\beta = 1/3$, and that Alpha's and Beta's annual incomes are \$48,000 is \$40,000, respectively.

(a) Suppose that Alpha is unwilling to contribute anything toward Joey's support, so that Beta must provide. What levels of x and y_B will Beta choose?

(b) Actually, Alpha, as well as Beta, is willing to contribute to Joey's support. What will be their equilibrium contributions to Joey's support?

(c) Find an allocation of the parents' incomes that will make them both happier than the one in (b).

(d) Determine the conditions that characterize the Pareto optimal allocations.

(e) Indicate some of the difficulties that a neutral third party (e.g., a judge) might encounter in attempting to implement some method for arriving at a Pareto optimal allocation of the parents' incomes.

Exercise 7. 100 men have access to a common grazing area. Each man can choose to own either no cows, one cow, or two cows to provide milk for his family. The more cows the grazing land is required to support, the lower is each cow's yield of milk; specifically, a man who owns x_i cows obtains

$$Q_i = (250 - x)x_i$$

quarts of milk per year, where $x = \sum_{j=1}^{100} x_j$ is the total number of cows in the grazing land supports. Each man wants to obtain as much milk as he can, but no man has the resources to own more than two cows.

(a) How many cows do you predict each man will own? (Justify your answer.)

(b) Assume that the men can make transfers of milk among themselves. Is your prediction in (a) Pareto efficient for the 100 men? If so, verify it. If not, find a pattern of cow ownership and transfer payments that yields a Pareto optimal allocation of milk to the men that makes everyone strictly better off than in (a).

(c) Now suppose that there are only two men whose cows share a common grazing area, and that each cowls daily yield of milk, in quarts, depends on how many cows in total are grazing according to the following table

Total cows grazing:	1	2	3	4
Each cowls daily yield:	8	5	3	2

Which allocations of milk are Pareto efficient and individually rational (i.e., such that each man is at least as well off as he would be by "unilateral" action)? What are all the patterns of cow ownership and transfer payments that will support these allocations? Determine all the core allocations of milk to the two men.

(d) For the situation described in part (c), answer all the questions posed in part (a).