Example 4

Example 4. Consider an agency problem in which $\pi(z) = z$, $u(w) = \sqrt{w}$, $c(e) = e^2/2$, and $\underline{u} = 1$. The set of feasible effort levels e is $[0, 1]$. Effort is not verifiable. Revenue X takes two values $x_1 = 4$, and $x_2 = 8$ with probabilities $p(e) = (2 - e)/3$ and $1 - p(e) = (1 + e)/3$, respectively. Identify the optimal contract.

Participation constraint:

$$\sqrt{w_1}\,(2 - e)/3 + \sqrt{w_2}\,(1 + e)/3 = 1 + e^2/2$$

Incentive Constraint

$$\tilde{e} = \arg\max_{e \in [0,1]} \mathbb{E}[W(e)] - c(e) = \frac{1}{3}(\sqrt{w_2} - \sqrt{w_1})$$

Example 4

Solving the system

$$\begin{aligned}\sqrt{w_1}\,(2-e)/3+\sqrt{w_2}\,(1+e)/3 &= 1+e^2/2\\ \tilde{e} &= \frac{1}{3}(\sqrt{w_2}-\sqrt{w_1})\end{aligned}$$

for $\sqrt{w_1}$ and $\sqrt{w_2}$, we get

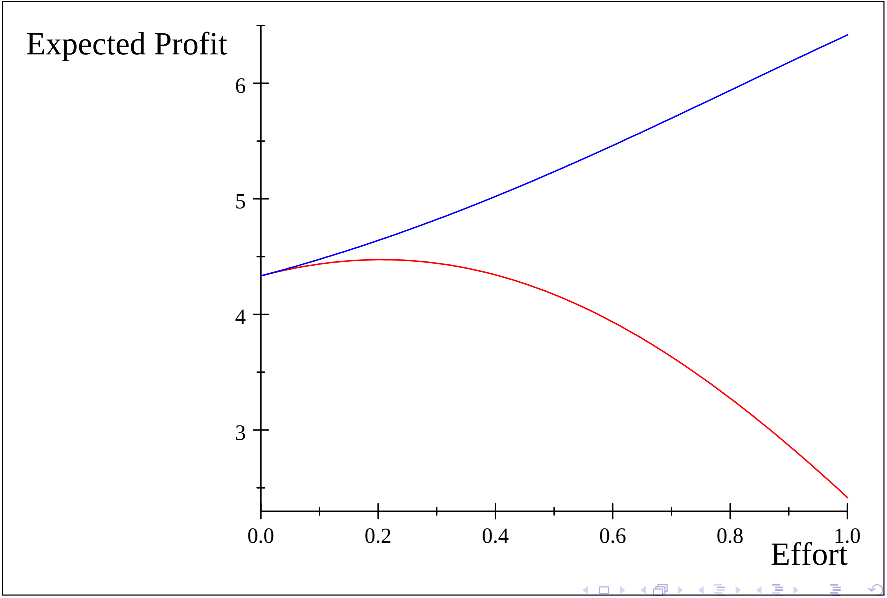
$$\begin{aligned}\sqrt{w_1} &= 1-\frac{1}{2}\tilde{e}^2-\tilde{e}\\ \sqrt{w_2} &= 1-\frac{1}{2}\tilde{e}^2+2\tilde{e}\end{aligned}$$

The optimal effort maximizes expected profit, $\mathbb{E}[X(\tilde{e})-W^*(\tilde{e})]$ i.e.,

$$\max_{\tilde{e}\in[0,1]} = \left(4-\left(1-\frac{1}{2}\tilde{e}^2-\tilde{e}\right)^2\right)\frac{2-\tilde{e}}{3}+\left(8-\left(1-\frac{1}{2}\tilde{e}^2+2\tilde{e}\right)^2\right)\frac{1+\tilde{e}}{3},$$

which solution is $\tilde{e}^*=0.20545$, leading to $\mathbb{E}[X(\tilde{e}^*)-W^*(\tilde{e}^*)]=4.4733$.

Example 4



Example 4

When effort is verifiable, the optimal wage contract is fixed and is given by the solution to the PC

$$\sqrt{\bar{w}} = 1 + e^2/2,$$

that is, $\bar{w}(e) = \left(1 + e^2/2\right)^2$, and the optimal effort maximizes $\mathbb{E}[X(e) - \bar{w}(e)]$, i.e., solves

$$\max_{e \in [0,1]} \left(4 - \left(1 - \frac{x^2}{2}\right)^2\right) \frac{2-x}{3} + \left(8 - \left(1 - \frac{x^2}{2}\right)^2\right) \frac{1+x}{3}$$

which solution is $e^* = 1$, leading to expected profit of 6.4167.