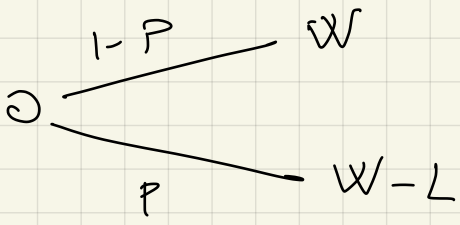
Competitive Insurance Markets

THE MARKET FOR INSURANCE.

(M. Rothschild and J. Stiglitz: Equilibrium in Competitive Insurance .... QJE (1976))

- A population of individuals face the risk of a wealth loss L with probability $p \in (0,1)$; that is, face the lottery



- Their preferences are represented by a Bernoulli utility function $u : \mathbb{R} \to \mathbb{R}$, such that $u' > 0$, $u'' < 0$.

- There is a competitive insurance market where firms offer policies (I, D), where

  I : Premium

  D : Deductible.

If an individual subscribes the policy (I, D), then he faces the lottery

$$(I, D) \quad \circ \begin{cases} \xrightarrow{1-p} \; W - I := x_n \\[2ex] \xrightarrow{\;\;p\;\;} \; W - I - D := x_a \end{cases}$$

Hence

$$Eu(I, D) = (1-p)\, u(x_n) + p\, u(x_a) := U(x_n, x_a)$$

Examples:

1. $(I, D) = (0, L)$ (No insurance)

2. $(I, D) = (I, 0)$ (Full insurance)

3. $(I, D) = (pL, 0)$ (Full insurance <u>fair</u> premium)

4. $(I, D) = (I, \frac{L}{2})$ (Partial insurance)

Typically, $D < 0$ is not allowed.

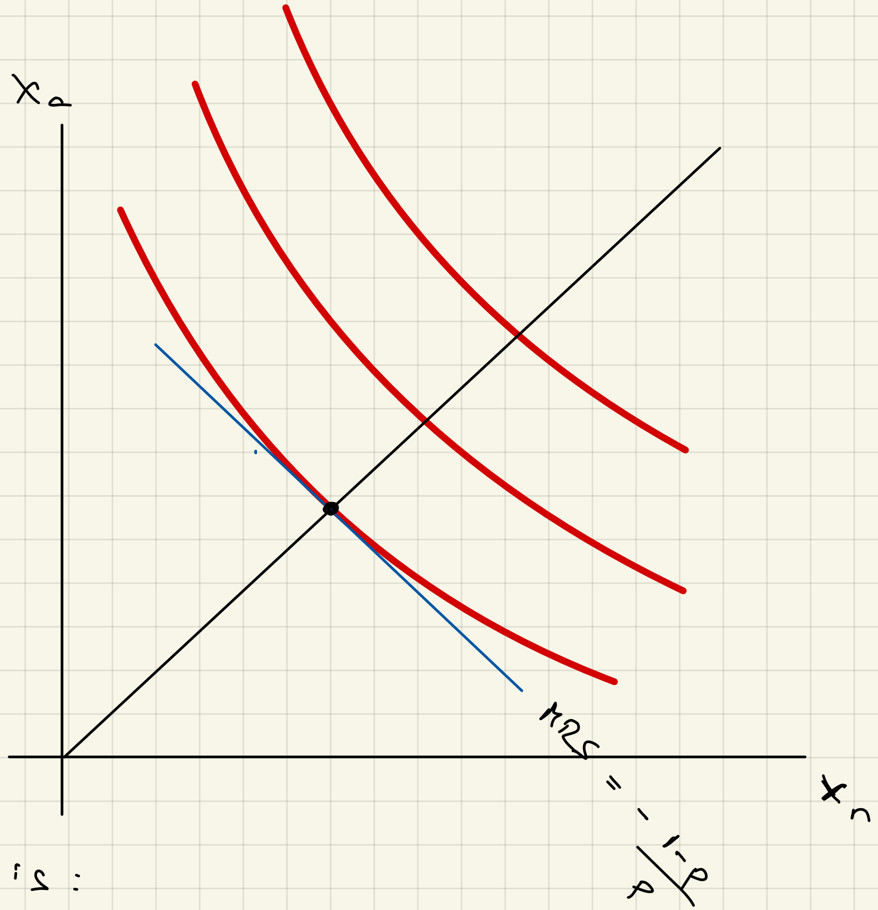
Preferences for insurance policies:

Let us calculate the $MRS(x_n, x_a)$:

$$U(x_n, x_a) = (1-p)\, u(x_n) + p\, u(x_a)$$

Hence

$$-\frac{dx_a}{dx_n} = MRS(x_n, x_a) = \frac{\frac{\partial U}{\partial x_n}}{\frac{\partial U}{\partial x_a}} = \frac{1-p}{p}\, \frac{u'(x_n)}{u'(x_a)}$$



Exercise: Check that $u' > 0$, $u'' < 0$ imply that indifference curves are convex, that is:

$$\frac{d^2 x_a}{d x_n^2} > 0.$$

FAIR ODDS POLICIES

$$x_n = W - I$$

$$x_a = W - I - D = x_n - D$$

$$I = p(L - D)$$

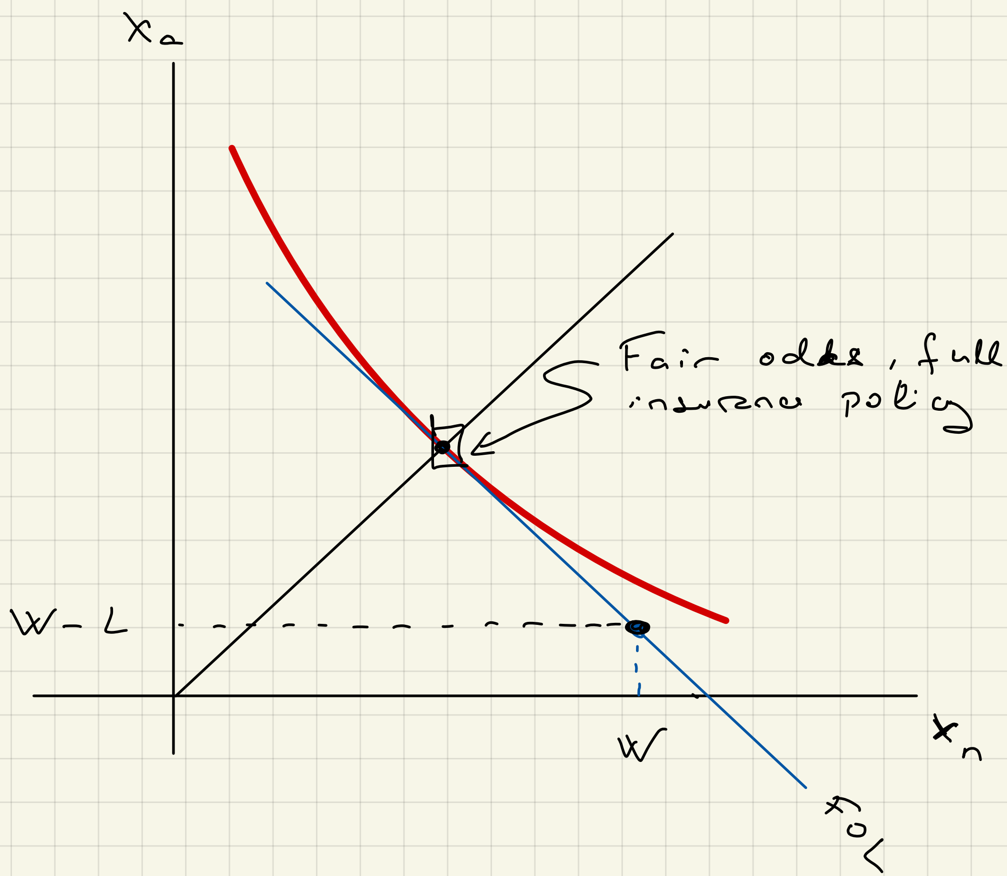
$$\Updownarrow$$

$$W - x_n = p\left[L - (x_n - x_a)\right]$$

$$\Updownarrow$$

FAIR ODDS LINE:
(FOL)

$$x_a = \left(\frac{W}{p} - L\right) - \frac{1-p}{p}\, x_n$$



CE :

In a CE a policy (I, D) is subscribed only if

(1) $I = p(L - D)$

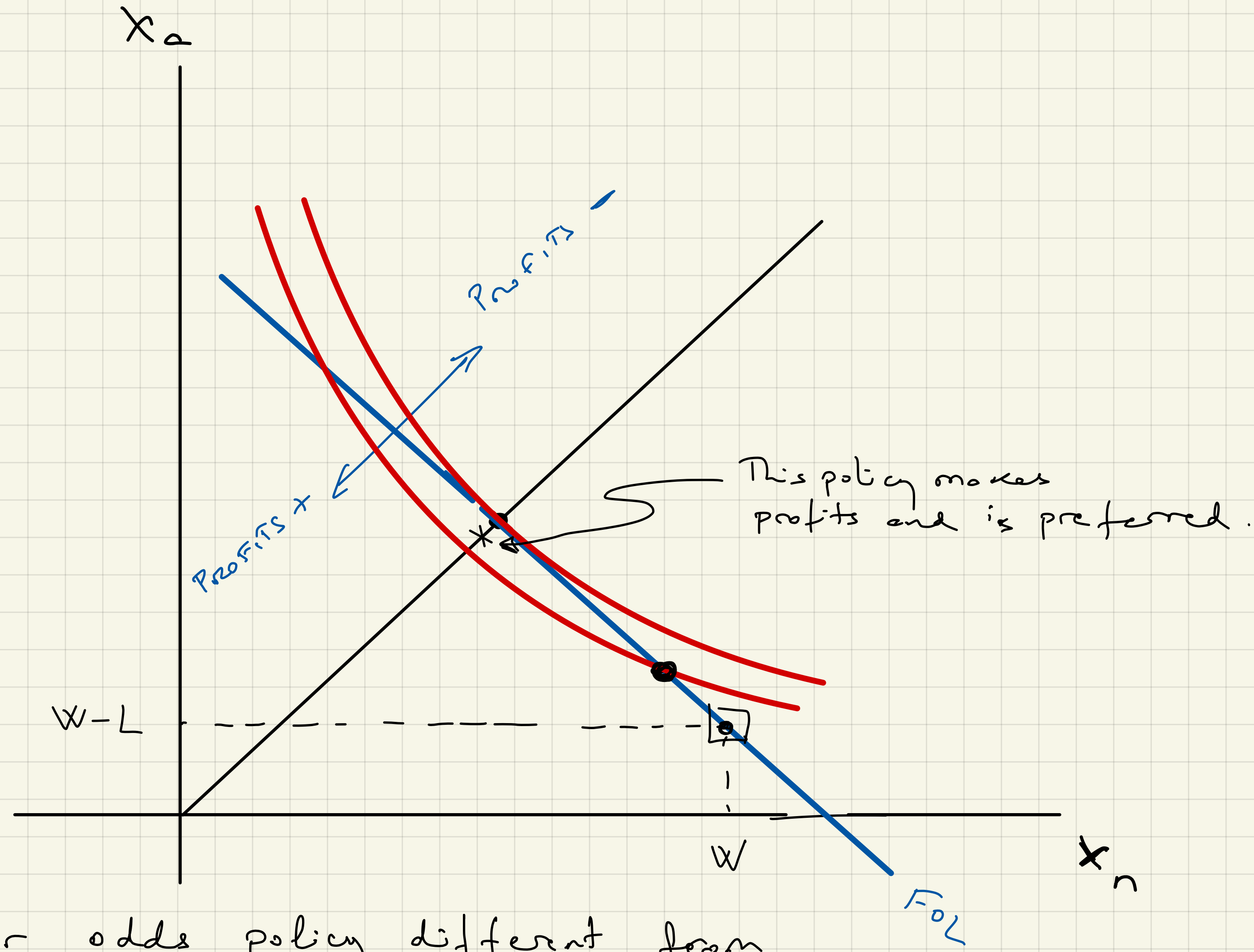
(2) $\nexists\ (\tilde{I}, \tilde{D})$ such that

- $\tilde{I} > p(L - \tilde{D})$
- $Eu(\tilde{I}, \tilde{D}) > Eu(I, D)$.

Proposition. In a CE all individuals subscribe the policy

$$(I^*, D^*) = (pL, 0).$$

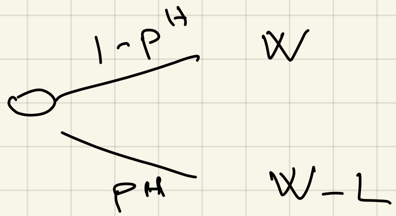
Proof



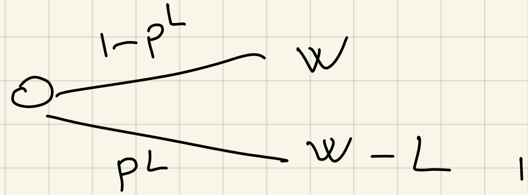
Any fair odds policy different from $(pL, 0)$ is upset by a policy $(pL+\varepsilon, 0)$ that makes profits.

Adverse Selection

Assume that a fraction $\lambda \in (0,1)$ of individuals face the risk



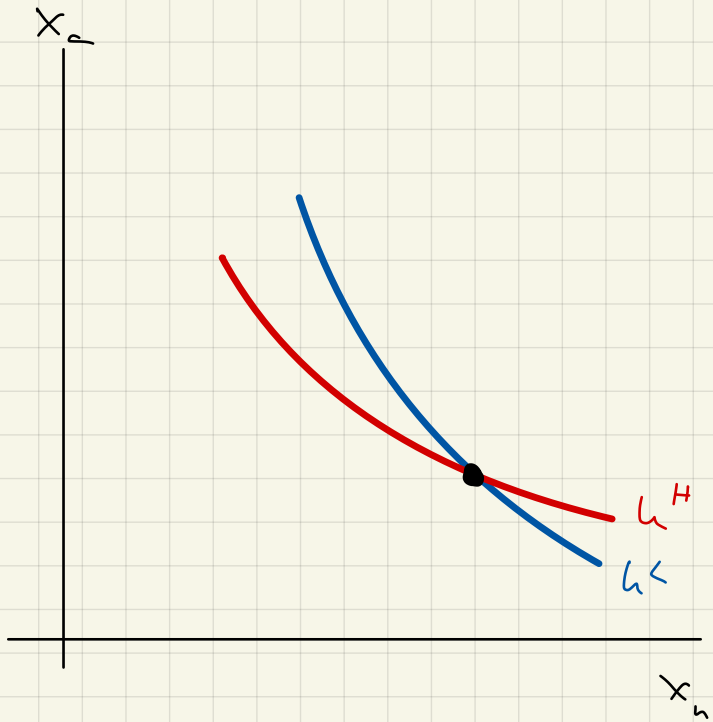
while the remaining fraction face the risk



where $0 < p^L < p^H$

Thus,

$$MRS_L(x_n, x_a) = \frac{1-p^L}{p^L}\frac{u'(x_n)}{u'(x_a)} > \frac{1-p^H}{p^H}\frac{u'(x_n)}{u'(x_a)} = MRS_H(x_n, x_a)$$



Note:

$$\frac{d}{dp}\left(\frac{1-p}{p}\right) = \frac{d}{dp}\left(\frac{1}{p}-1\right) = -\frac{1}{p^2} < 0.$$

If insurance companies could recognize the individual of either type, then in a CE they will offer to each type her full insurance fair odds policy.

What if insurance companies cannot distinguish between high and low risk individuals?

Which policies will they offer in a CE?

Will they offer a single (pooling) policy to both types?

<u>Proposition</u> A CE in which the same policy is subscribed by both types (i.e., a pooling <u>equilibrium</u>) does not exist.

$x_a = W - I - D$

$x_n = W - I$

$I = p(L-D)$

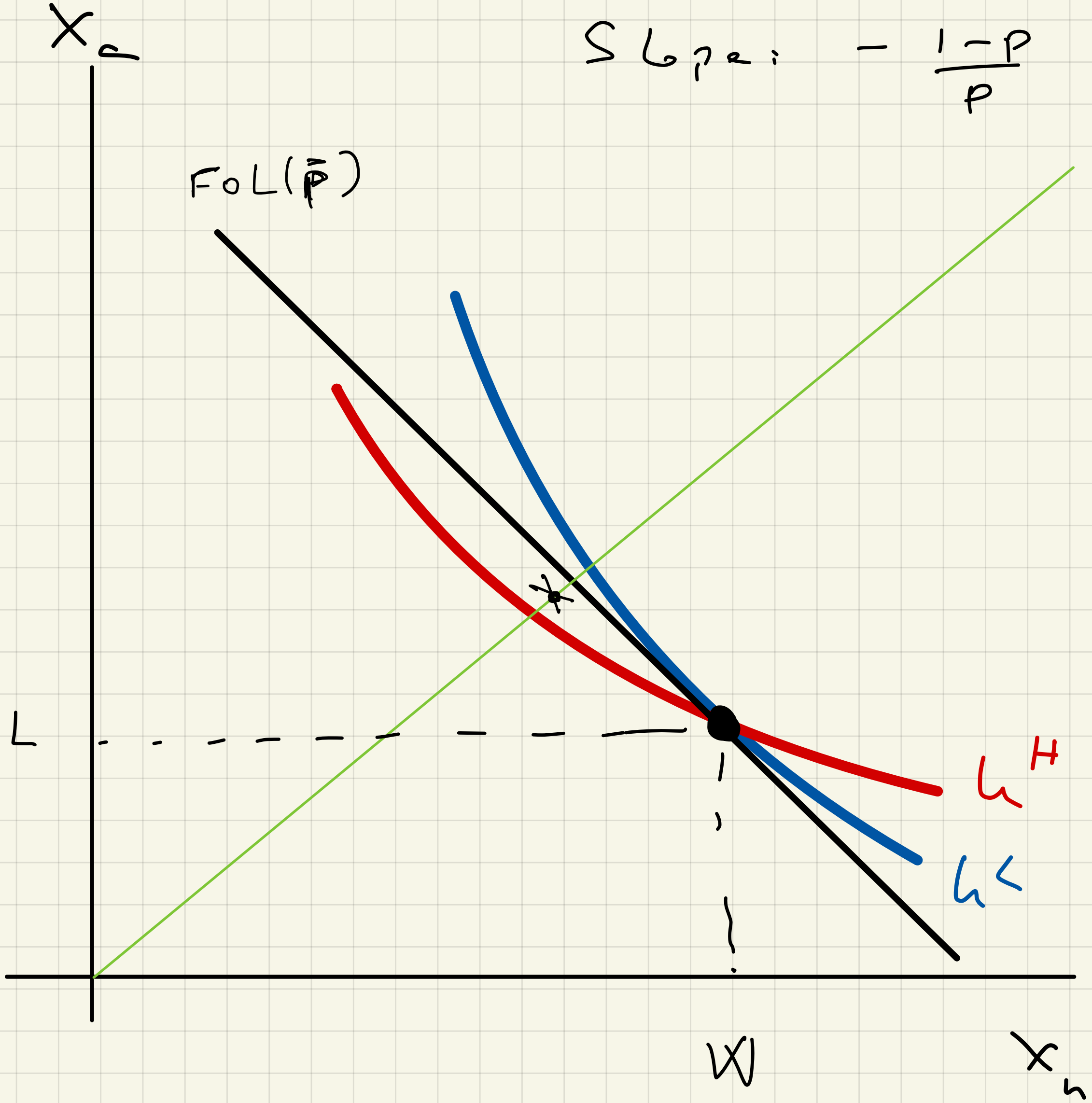
Slope: $-\frac{1-p}{p}$

Proof

(a) Offering the single policy $(0, L)$ is not a CE.

A company offering the policy $(p^H L + \varepsilon, 0)$ will attract all the high risk individuals and make positive profits.

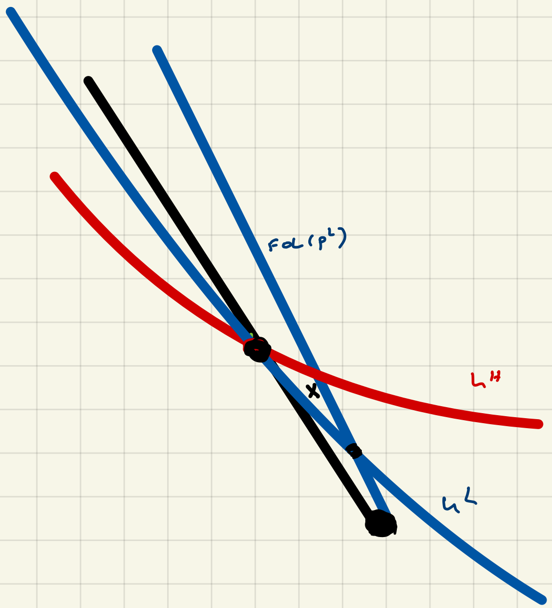
(Note. It may also attract the low risk individuals.)

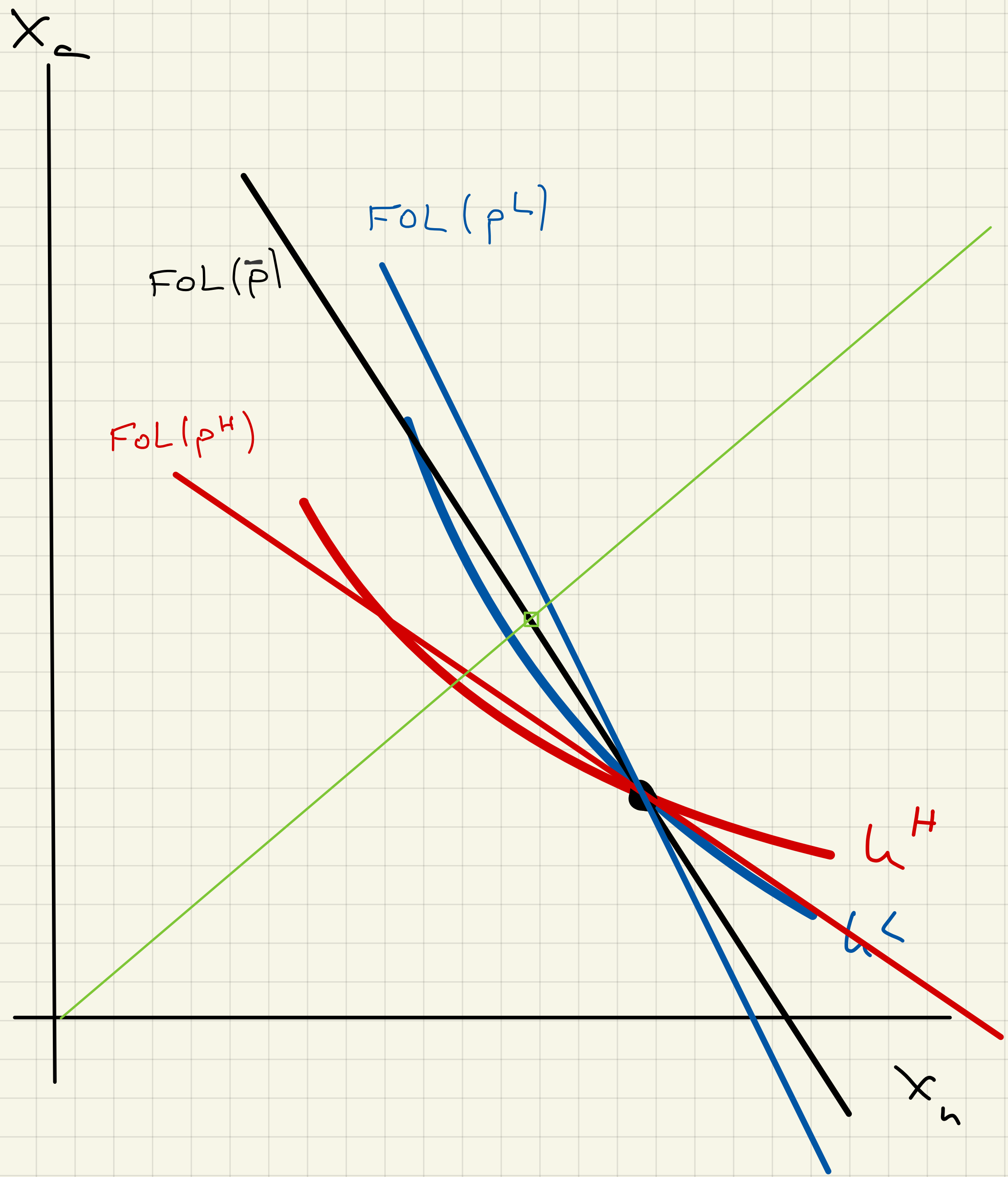


(b) Any other fair odds policy $(p(L-D), D)$ with

$$\bar{p} = \lambda p^H + (1-\lambda) p^L,$$

will be upset by a policy aimed to attracting the low risk individuals



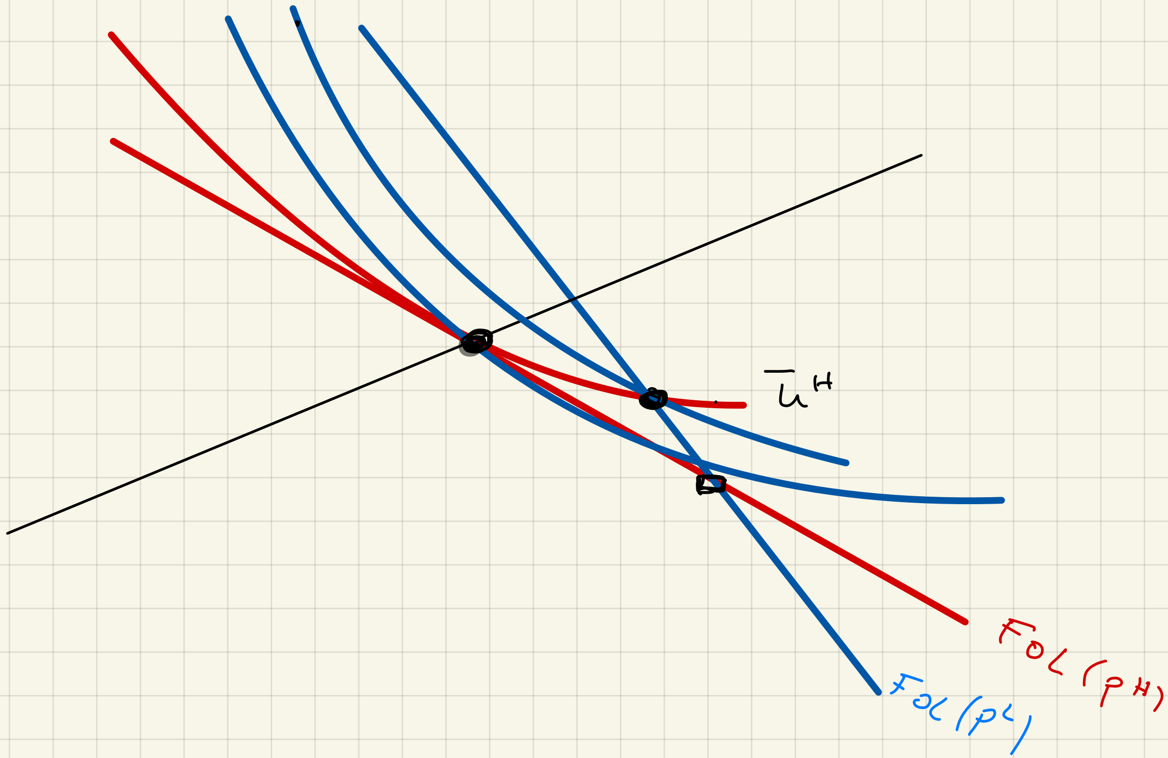


A SEPARATING EQUILIBRIUM

$\{(I^H, D^H), (I^L, D^L)\}$, ① $I^H = p^H L$, $D^H = 0$

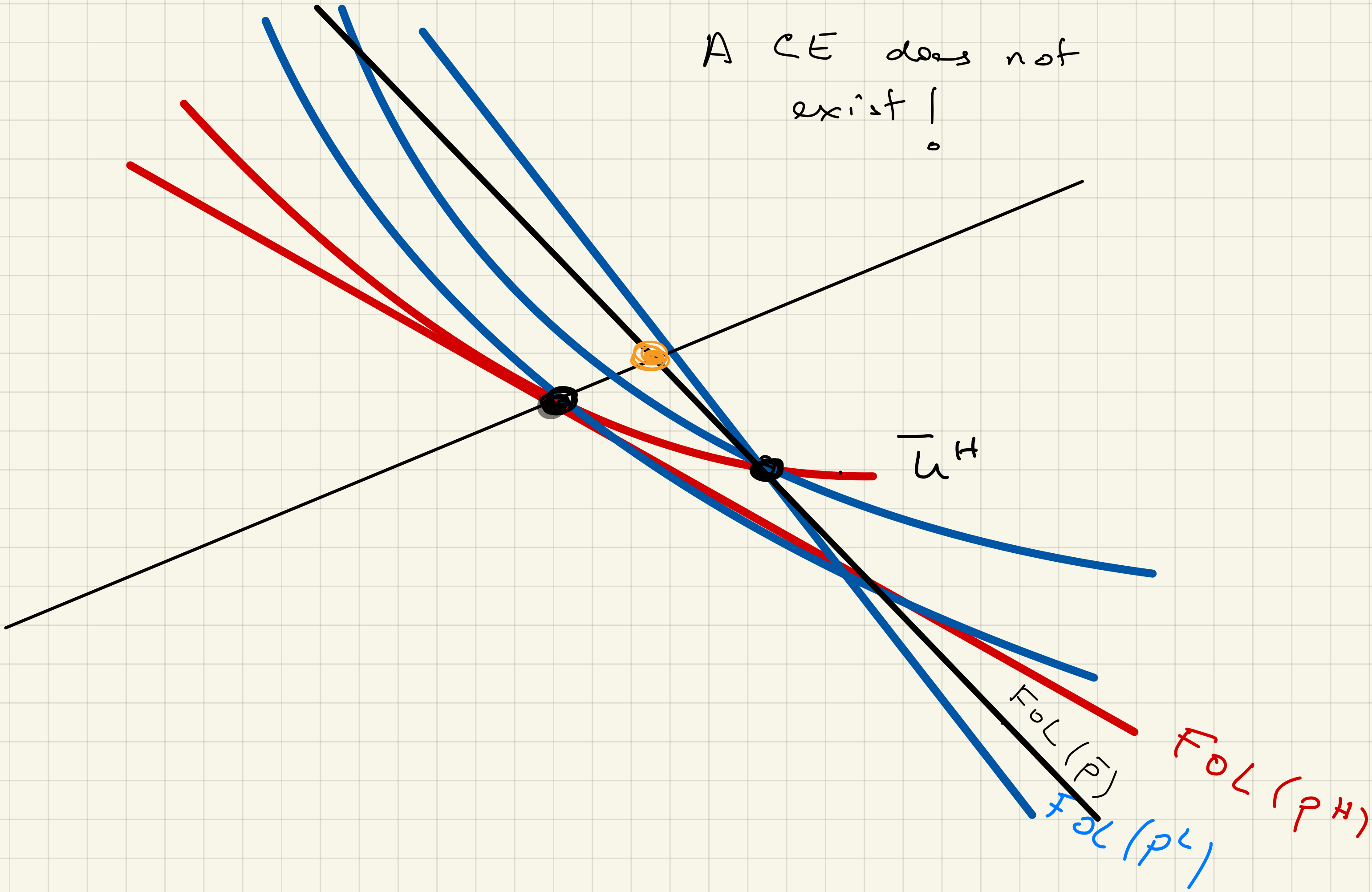
② $I^L = p^L (L - D^L)$

$E_H u(I^L, D^L) := p^H u(W - I^L - D^L) + (1-p^H) u(W - I^L) = E_H u(I^H, D^H) = u(W - I^H)$.



For this menu to be a CE, λ must be sufficiently large that.

$$u(W - \bar{p}L) \leq p^L u(W - I^L - D^L) + (1 - p^L)\, u(W - I^L)$$



$W = L = 1, \quad u(x) = \sqrt{x}, \quad p^L = \frac{1}{4}, \quad p^H = \frac{1}{2}, \quad \lambda \in (0,1).$

WHEN TYPES ARE OBSERVABLE, THE CE POLICIES ARE:

$$(I^H, D^H) = \left(\frac{1}{2}, 0\right), \quad (I^L, D^L) = \left(\frac{1}{4}, 0\right)$$

THE <u>POOLING POLICY</u> IS $(\bar{I}, \bar{D}) = (\bar{p}(\lambda) L, 0)$,

WHERE:

$$\bar{p}(\lambda) = \frac{\lambda}{2} + \frac{(1-\lambda)}{4} = \frac{1+\lambda}{4}.$$

FOR $\lambda = 1/4$, FOR EXAMPLE, $\bar{p}(1/4) = \frac{5}{16}$, AND

$$(\bar{I}, \bar{D}) = \left(\frac{5}{16}, 0\right)$$

WITH THIS POLICY, THE EXPECTED UTILITIES OF AGENTS ARE

$$\bar{U}_H = \bar{U}_L = \frac{\sqrt{11}}{4} \simeq .83$$

BUT THIS POLICY IS "DESTABILIZED" BY, E.G., THE POLICY

$$\left(\tilde{I}^L, \tilde{D}^L\right) = \left(\frac{1}{8}, \frac{1}{2}\right).$$

THE EXPECTED UTILITIES OF AGENTS THAT SUBSCRIBE THIS POLICY ARE

$$\tilde{U}_L = \frac{3}{4}\sqrt{\frac{7}{8}} + \frac{1}{4}\sqrt{3} \simeq .854$$

$$\tilde{U}_H = \frac{1}{2}\sqrt{\frac{7}{8}} + \frac{1}{2}\sqrt{\frac{3}{8}} \simeq .773.$$

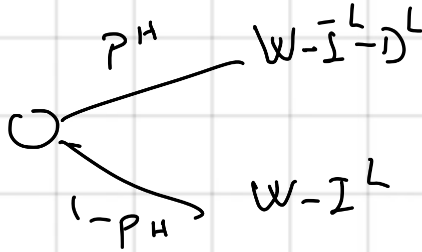
HENCE LOW RISK INDIVIDUALS PREFER THIS POLICY TO THE POOLING POLICY, WHILE HIGH RISK INDIVIDUALS PREFER THE POOLING POLICY.

THE SEPARATING POLICIES ARE:

$(I^H, D^H) = \left(\frac{1}{2}, 0\right)$, $(I^L, D^L) = \left(p^L (L - D^*), D^* \right)$ WHERE D $I^L = \frac{1-D}{4}$

SOLVES:

$$\frac{1}{2}\sqrt{1 - \frac{1-D}{4}} + \frac{1}{2}\sqrt{1 - \frac{1-D}{4} - D} = \frac{1}{\sqrt{2}}$$



WHERE $\frac{1}{\sqrt{2}} = U_H^S$ IS THE EXPECTED UTILITY OF A HIGH RISK INDIVIDUAL WHO SUBSCRIBES THE POLICY (I^H, D^H). SOLVING THIS EQUATION WE GET

$$\boxed{D^* = \sqrt{3} - 1 \simeq .732.}$$

$$I^L = \frac{1-D^*}{4} = \frac{1-(\sqrt{3}-1)}{4} = \frac{2-\sqrt{3}}{4}$$

THE SEPARATING "MENU" FORMS A CE PROVIDED THE (EXP.) ULITY OF LOW RISK INDIVIDUALS OF THE POOLING POLICY, $(\bar{p}(\lambda)L, 0)$, WHERE

$$\bar{p}(\lambda) = \frac{\lambda}{2} + \frac{1-\lambda}{4} = \frac{1+\lambda}{4},$$

WHICH IS

$$u\left(1 - \frac{1+\lambda}{4}\right) = u\left(\frac{3-\lambda}{4}\right) = \frac{\sqrt{3-\lambda}}{2},$$

IS LESS THAN THE EXPECTED UTILITY OF THE POLICY (I^L, D^L),

$$\begin{aligned} Eu(I^L, D^L) &= p^L u(1 - I^L - D^L) + (1-p^L)\, u(1 - I^L) \\ &= \frac{1}{4}\sqrt{1 - \frac{2-\sqrt{3}}{4} - (\sqrt{3}-1)} + \frac{3}{4}\sqrt{1 - \frac{2-\sqrt{3}}{4}} \\ &= \frac{\sqrt{6}}{8}(\sqrt{3}+1). \end{aligned}$$

THAT IS

$$\frac{\sqrt{6}}{8}(\sqrt{3}+1) \geq \frac{\sqrt{3-\lambda}}{2} \iff \lambda \geq \frac{3}{4}(2-\sqrt{3}) \simeq 0.2$$

NOTE: IF THE POOLING POLICY (WHICH IS PREFERED TO THE SEPARATING POLICIES BY THE HIGH RISK INDIVIDUALS) IS PREFERED TO THE SEPARATING POLICY BY THE LOW RISK INDIVIDUAL (AS WELL) THEN:

- THE POOLING POLICY IS PARETO SUPERIOR TO THE SEPARATING POLICIES, AND
- THE POLICY $(\bar{p}(\lambda)+\varepsilon, 0)$, FOR $\varepsilon > 0$ SMALL, DE-STABILIZES THE SEPARATING POLICIES — THAT IS, THE SEPARATING POLICIES DO NOT FORM A CE.

...BES THE

THUS, WHEN THE SEPARATING MENU IS <u>NOT</u> A CE ....

MAKING THE POOLING POLICY <u>MANDATORY</u> MAKES EVERYONE BETTER OFF.

(Ex. 3 IN LIST 2 IS ANALOGOUS.)