Time and Uncertainty in Competitive Economies

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Time

Consider an economy

$$[Y_1, ..., Y_m, (u_1, \bar{x}_1, \theta_1), ..., (u_n, \bar{x}_n, \theta_n)]$$

that operates over T consecutive dates. Since in this economy the number of goods is IT, let us assume that $\mathbb{R}_+^{IT}$ is the *commodity space*, so that

$$Y_j \subset \mathbb{R}^{IT}$$

for all $j \in \{1, ..., m\}$, and

$$u_i : \mathbb{R}_+^{IT} \to \mathbb{R}, \ \bar{x}_i \in \mathbb{R}_+^{IT}$$

for all $i \in \{1, ..., n\}$.

Time

Issues:

$\triangleright$Are goods perishable? Are they storable (perhaps at a cost)?

$\triangleright$Is T finite or infinite? (If it is, we have infinitely many commodities)

$\triangleright$Is there technological progress?

(For example, $Y_j = Y_j^1 \times ... \times Y_j^T$, where $Y_j^t \subset Y_j^{t+1}$.)

$\triangleright$Does production take time?

(Which would raise the issue of investment.)

$\triangleright$What are the dynamic features of preferences?

(If no assumptions are made, no conclusions about consumption smoothing, habits or persistence are derived. For example, what if

$$u(x) = \sum_{t=1}^{T} \delta^{t-1} v(x_t),$$

where $v : \mathbb{R}_+^l \to \mathbb{R}$.)

Time

Consider a pure exchange economy operating over T consecutive dates, in which goods are perishable. Consumers' preferences and initial endowments in this economy are described by the profile

$$[(u_1, \bar{x}_1), ..., (u_n, \bar{x}_n)].$$

where for $i \in \{1, ..., n\}$, $\bar{x}_i \in \mathbb{R}_+^{lT}$ and $u_i : \mathbb{R}_+^{lT} \to \mathbb{R}$.

In an *Arrow-Debreu economy* there are (future) markets for all goods. Thus, if the market prices are $p = (p_1, ...p_T) \in \mathbb{R}_+^{lT}$, then the budget set of a consumer endowed with $\bar{x} \in \mathbb{R}_+^{lT}$ is

$$B(p, \bar{x}) = \left\{ x \in \mathbb{R}_+^{lT} \mid \sum_{t=1}^{T} p_t (x_t - \bar{x}_t) \leq 0 \right\}.$$

(What if goods are storable?)

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A (Arrow-Debreu) CE for this economy is a collection (p^*, x^*), where $p^* \in \mathbb{R}_+^{IT}$ is a *price system* and $x^* = (x_1^*, ..., x_n^*) \in \mathbb{R}_+^{ITn}$ is an *allocation*, satisfying:

1. For all $i \in \{1, ..., n\}$,

$$x_i^* \in x_i(p^*) := \arg \max_{B(p^*, \bar{x}_i)} u_i(x).$$

2. For all (k, t),

$$\sum_{i=1}^{n} (x_{itk}^* - \bar{x}_{itk}) \leq 0 \text{ (with equality if } p_{tk}^* > 0).$$

Arrow-Debreu CE allocations are PO (FWT)

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In real economies there are future markets for some but not all goods. Moreover, there are *spot* markets for all goods, as well as *credit* markets whereby consumers can borrow or lend.

In the Arrow-Debreu (static) setting, after the operation of future markets no further transactions are needed. Thus, the theory leaves no role for spot or credit *transactions*.

(But this implication is a technical artifact. In real economies there are inflows and outflows of agents, which may justify the existence of transactions in spot and future markets every period.)

- What are the implications of missing (future) markets?
- What is the role of credit and spot market transactions in the intertemporal allocation of consumption?

Assume instead that there are no future markets in the economy, but there are spot and credit markets operating each date.

We refer to economies with this market structure as a *Radner economies.*

In a Radner economy, if the spot prices are $\hat{p} = (\hat{p}_1 \ldots, \hat{p}_T)$ and the interest rates are $r = (r_1 \ldots, r_{T-1})$, then the budget set of a consumer endowed with $\bar{x} \in \mathbb{R}_+^{IT}$ is

$$
\begin{aligned}
\hat{B}(\hat{p}, r, \bar{x}) = \quad & \{(x, b) \in \mathbb{R}_+^{IT} \times \mathbb{R}^{T-1} \mid \\
& \hat{p}_1(x_1 - \bar{x}_1) \leq b_1, \\
& \hat{p}_t(x_t - \bar{x}_t) + (1 + r_{t-1})b_{t-1} \leq b_t, \text{ for } 1 < t < T, \\
& p_T(x_T - \bar{x}_T) + (1 + r_{T-1})b_{T-1} \leq 0\}.
\end{aligned}
$$

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A (Radner) CE of this economy is a profile $(\hat{p}^*, r^*, b^*, x^*)$, where

$$(\hat{p}^*, r^*) \in \mathbb{R}_+^{lT} \times \mathbb{R}^{T-1}$$

is a (sequence) of spot prices and interest rates *expectations*,

$$b^* = (b_1^*, ..., b_n^*) \in \mathbb{R}^{(T-1)n}$$

is a profile of *credit plans*, and

$$x^* = (x_1^*, ..., x_n^*) \in \mathbb{R}_+^{lTn}$$

is an allocation, satisfying:

1. $(x_i^*, b_i^*) \in \arg \max_{\hat{B}(\hat{p}^*, r^*, \bar{x}_i)} u_i(x)$, $\forall i \in \{1, ..., n\}$.
2. $\sum_{i=1}^n (x_{itk}^* - \bar{x}_{itk}) \leq 0$ (with equality if $\hat{p}_{tk}^* > 0$), $\forall (k, t)$.
3. $\sum_{i=1}^n b_{it}^* = 0$, $\forall t < T$.

Are Radner CE allocations Pareto optimal?

Proposition. If $(\hat{p}^*, r^*, b^*, x^*)$ is a Radner CE, then (p^*, x^*), where $p_T^* = \hat{p}_T^*$, and for $t < T$,

$$p_t^* = \hat{p}_t^* \prod_{s=t}^{T-1} (1 + r^*),$$

is an Arrow-Debreu CE. Hence a Radner CE allocation is Pareto optimal.

Proof. For simplicity, assume $T = 2$. We show that
(1) $x_i^* \in B(p^*, \bar{x}_i)$, and

(2) $B(p^*, \bar{x}_i) \subset \hat{B}_x(\hat{p}^*, r^*, \bar{x}_i)$, where

$$\hat{B}_x(\hat{p}^*, r^*, \bar{x}_i) = \{x \in \mathbb{R}_+^{lT} \mid (x, b) \in \hat{B}(\hat{p}^*, r^*, \bar{x}_i) \text{ for some } b \in \mathbb{R}^{T-1}\}.$$

Hence, since (x_i^*, b_i^*) maximizes u_i on $\hat{B}(\hat{p}^*, r^*, \bar{x}_i)$, x_i^* also maximizes u_i on $B(p^*, \bar{x}_i)$.

Thus, $x_i^* \in x_i(p^*)$, and markets clear at prices p^*.

Therefore (p^*, x^*) is an Arrow-Debreu CE.

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(1) $x_i^* \in B(p^*, \bar{x}_i)$. Since $\hat{p}_1^*(x_{i1}^* - \bar{x}_{i1}) \leq b_i^*$,

$$\begin{aligned} p_1^*(x_{i1}^* - \bar{x}_{i1}) + p_2^*(x_{i2}^* - \bar{x}_{i2}) &= (1+r_1^*)\hat{p}_1^*(x_{i1}^* - \bar{x}_{i1}) + \hat{p}_2^*(x_{i2}^* - \bar{x}_{i2}) \\ &\leq (1+r_1^*)b_i - (1+r_1^*)b_i^* \\ &= 0. \end{aligned}$$

(2) $B(p^*, \bar{x}_i) \subset \hat{B}(\hat{p}^*, r^*, \bar{x}_i)$. Let $x_i \in B(p^*, \bar{x}_i)$. We show that $(x_i, b_i) \in \hat{B}(\hat{p}^*, r^*, \bar{x}_i)$, where $b_i = \hat{p}_1^*(x_{i1} - \bar{x}_{i1})$.

$$\begin{aligned} \hat{p}_2^*(x_{i2} - \bar{x}_{i2}) + (1+r_1^*)b_i &= \hat{p}_2^*(x_{i2}^* - \bar{x}_{i2}) + (1+r_1^*)\hat{p}_1^*(x_{i1}^* - \bar{x}_{i1}) \\ &= p_1^*(x_{i1}^* - \bar{x}_{i1}) + p_2^*(x_{i2}^* - \bar{x}_{i2}) \\ &\leq 0. \ \square \end{aligned}$$

Time

Let us assume that u_i is increasing, so that budget constrains are binding. If $x \in B_i(\hat{p}^*, r^*)$, then $\exists b \in \mathbb{R}$ such that

$$p_1(x_1 - \bar{x}_{i1}) = b$$
$$p_2(x_2 - \bar{x}_{i2}) + (1 + r_1)b = 0$$

which implies

$$(1 + r_1)p_1(x_1 - \bar{x}_{i1}) + p_2(x_2 - \bar{x}_{i2}) = 0,$$

i.e., future value of consumption equal to future value of endowments.
Also, since $1 + r_1 > 0$ (why?), we may write

$$p_1(x_1 - \bar{x}_{i1}) + \frac{p_2}{1 + r_1}(x_2 - \bar{x}_{i2}) = 0,$$

i.e., present value of consumption equal to present value of endowments.