Final Exam (April 2010)

1. (40 points) Ann and Bob are the only inhabitants of an island. Their production activities are fishing, and collecting fruit. Ann's dietary preferences for fish (x) and fruit (y) are described by the utility function $u^A(x,y) = x + 2\ln y$. Bob simply cares about the total number of calories of his diet – one pound of fish has twice as many calories as one pound of fruit. Thus, his preferences are described by the utility function $u^B(x,y) = 2x + y$. Ann is very able climbing trees and can collect as much as 12 pounds of fruit a day, but she cannot swim, and is therefore unable to fish. Bob is an able fisherman whose daily catch is 6 pounds of fish; however, he suffers from dizziness (vertigo) and is unable to climb up a tree to collect fruit.

(a) Determine the sets of feasible and Pareto optimal allocations.

(b) Calculate the competitive equilibrium price vectors and allocations.

(c) Determine the core of the economy.

(d) Carol, another survivor of a shipwreck, arrives at the island. Her production abilities and preferences are identical to those of Ann. Calculate the competitive equilibrium of the economy formed by Ann, Bob and Carol. Also determine whether the core satisfies equal treatment.

Answer any two of the following three questions.

2. (20 points) There are two fishermen who have free access to the local lake. Each must choose how many days a week to devote to fishing, z_i. The total catch of fish obtained by each fisherman is $z_i\,(3 - \bar{z})$, where z_i is the number of days devoted to fishing by the individual, and $\bar{z} = \frac{1}{2}(z_1 + z_2)$ is the average number of days the two men fish during the weak. Their preferences for fish and leisure are identical and are described by the utility function $u(x,y) = x + y$, where x is the pounds of fish consumed, and y is the number of days of leisure during the week. Naturally, each fisherman has 7 days a week for fishing and leisure activities.

(a) Calculate how each fisherman will allocate his time to leisure and fishing.

(b) Determine whether the allocation (a) is Pareto optimal by identifying the socially optimal average number of days the two men should fish.

3. (20 points) Robinson Crusoe is the single inhabitant of an island. His preferences for weekly leisure (x) and food (y) consumption are represented by the utility function $u(x,y) = xy^2$. He has 32 hours a week to distribute between leisure and labor, and no food. There is a firm own by Robinson that produces food using labor according to the production function $f(z) = \sqrt{z}$. Calculate the competitive equilibrium and show that it is Pareto optimal.

4. (20 points) There is only one perishable commodity available for consumption "today" and "tomorrow." There is uncertainty about the state of nature tomorrow, which may be

either Y or Z. There are two consumers in the economy, Ms. One and Ms. Two, who must decide their consumption streams (x, y, z), where x, y and z denote consumption today, and tomorrow at Y and Z, respectively. Ms. One's preferences are described by the utility function $u_1(x, y, z) = xy + 2xz$; she owns 10 units of the good today, and she will own 20 units of the good tomorrow if the state in Y and none if the state is Z. Ms. Two's preferences are described by the utility function $u_2(x, y, z) = 2xy + xz$; she owns 10 units of the good today, and she will own 20 units of the good tomorrow if the state is Z, and none if the state is Y.

(a) If Ms. One and Ms. Two engage in borrowing-and-lending in order to arrive at more desirable consumption streams than they are endowed with (i.e., there is a competitive credit market), what will be the equilibrium consumption streams? Will this allocation be Pareto optimal?

(b) Now suppose that in addition to the credit market, one may *bet* today on the occurrence of Y; i.e., one may buy or sell at the market price (q) contracts promising to pay one euro if Y occurs. Determine whether the new competitive equilibrium is Pareto optimal.