Final Exam (March 2009)

1. (40) Answer any four of the following questions:

1.1. Verify that the following statement is true (with a proof of some sort) or provide a counterexample: If some consumer has a lexicographic preference, then a competitive equilibrium will not exist.

1.2. Consider a two-good two-person pure exchange economy where each person's preference is represented by a utility function of the form $u^i(x, y) = \min\{\alpha_i x, y\}$, $\alpha_i \in \mathbb{R}_+$. Show that both the competitive price ratio and the competitive allocation are generically unique. (Genericity is relative to the set of initial allocations, $\mathbb{R}^4_+$.)

1.3. Calculate the competitive equilibria and the set of Pareto optimal allocations of the economy $[(u_1, (12, 0)), (u_2, (0, 4))]$, where $u_1(x, y) = xy$ and $u_2(x, y) = y + 2 \ln x$.

1.4. Determine whether the allocations $[(8, 8), (4, 4), (8, 8)]$ and $[(9, 9), (4, 4), (7, 7)]$ are in the core of the economy $[(u_1, (12, 4)), (u_2, (4, 4)), (u_3, (4, 12))]$, where $u_i(x, y) = xy$ for $i \in \{1, 2, 3\}$.

1.5. Consider an exchange economy where there is uncertainty about which of two possible states of the world, sunny (S) or cloudy (C) will occur. Consumption takes place after the uncertainty resolves. No contingent market operates in this economy, but there are spot markets and two securities, s_1, which pays \$1 in state S and noting in state C, and s_2, which pays \$2 in each state. Determine whether the competitive equilibrium allocations are Pareto Optimal.

1.6. Let $E = [(u_i, (0, \bar{y}_i))_{i=1}^n, f]$ be an economy where x is public good and y is private good, each u_i is differentiable, increasing in both arguments and strictly concave, and f is a differentiable, increasing, and strictly concave production function that allows to produce public good using private good as input. Show that in this economy the amount of public good provided under voluntary contributions is less than what would not be optimal.

2. (20) Ann and Bob are the only inhabitants of an island. Their production activities are fishing, and producing cloth using palm tree leaves. Bob's preferences for daily consumption of fish (x), and cloth (y), which he uses as clothing, are described by the utility function $u^B(x, y) = xy^2$. He is an able fisherman, whose daily catch is 3 pounds of fish; however, he does not know the technology for producing cloth. Ann only cares about her consumption of fish – the island enjoys a benign weather. She knows a technology that allows her to produce one square yard of cloth a day; however, she is a lousy fisher: it takes her an entire day to catch a single pound of fish.

(a) Determine the sets of feasible and Pareto optimal allocations.

(b) Calculate the economy's competitive equilibrium prices and allocations. (Set $p_x = 1$ and $p_y = p$. Keep in mind that Ann must decide how to allocate her day to fishing and producing cloth, and that she is a price-taker.) Assume that Bob discovers a technology that allows him to produce two square yards of cloth per day; how would this discovery affect the competitive equilibrium?

3. (20) Ann, Bob, and Carol are housemates. They must determine what level of cable TV service $x \in \Re_+$ they will subscribe to. Their preferences are described by utility functions of the form $u_i(x, y_i) = y_i + \alpha_i \sqrt{x}$, where y_i denotes the euros available to spend on other goods. The values of α for Ann, Bob, and Carol are $\alpha_A = \alpha_B = 6$, and $\alpha_C = 8$, respectively. Each housemate is endowed with 100 euros. The service fee 2 euros a unit.

(a) Calculate the service level that will result if it is decided by voluntary contribution and the service level that is consistent with Pareto optimality (assuming $y_i > 0$, for $i = A, B, C$).

(b) Suppose that they adopt the following procedure to decide the service level x and how to split the cost: Each housemate announces a service level $s_i \in \Re$; then they purchase the service level $x = s_A + s_B + s_C$, and each pays $\tau_A = (2 + s_B - s_C)x$, $\tau_B = (2 + s_C - s_A)x$, and $\tau_C = (2 + s_A - s_B)x$. Calculate the Nash equilibrium allocation, and determine whether it is Pareto optimal.