Final Exam (2007-08)

1. (30) Consider a two-person two-good pure exchange economy described by: $u_1(x,y) = yx$, $u_2(x,y) = 2x + y$, $\omega^1 = (4,2)$, and $\omega^2 = (0,2)$.

(a) Identify and graph the set of Pareto optimal allocations and the core of the economy.

(b) Calculate the set of competitive equilibrium prices and allocations.

(c) Assume now that there is a third consumer identical to consumer 2. Determine whether the core of this three-person economy satisfies the equal treatment property – that is, either proof that all core allocations assign the same bundle to consumers 2 and 3, or find a core allocation which does not assign the same bundle to consumers 2 and 3.

2. (20) There is a single perishable commodity (x) available for consumption, and two time periods, today ($t = 0$) and tomorrow ($t = 1$). The state of nature tomorrow is uncertain: it may be either sunny (s) or cloudy (c). There are two consumers whose endowments are $\omega^1 = (10, 0, 20)$ and $\omega^2 = (10, 20, 0)$, respectively, and whose preferences are represented by the utility function $u_i(x_i^0, x_i^s, x_i^c) = x_i^0 + \frac{1}{4}\ln(1 + x_i^s) + \frac{3}{4}\ln(1 + x_i^c)$, $i = 1, 2$. Normalize to one the spot market price of good x; i.e., $p_\tau = 1$ for $\tau \in \{0, s, c\}$.

(a) Assume that there is only a credit market where consumers can either borrow or lend. Denote by y_i the amount that consumer i lends (or borrows if $y_i < 0$), and let r be the rate of interest. Write down the budget constraints of consumers 1 and 2. What will be the equilibrium consumption stream of each consumer? Is this allocation Pareto optimal? Is this market structure complete?

(b) Now assume that in addition to the credit market there is another market in which one can buy or sell insurance against the occurrence of s – a unit of this insurance is a contract by which the seller agrees to pay the buyer a unit of consumption if tomorrow is sunny. Denote by z_i the amount of insurance that consumer i buys (or sells if $z_i < 0$), and let q be the insurance premium (the price of a unit of insurance). Write down the budget constraints of consumers 1 and 2. Note that since consumers' preferences are monotonic, these budget constraints are binding. Use this constrains to calculate the consumers' indirect utilities as functions of how much a consumer lends and how many units of insurance he buys, $v_i(y_i, z_i)$.

(c) In the situation described in (b), what will be the equilibrium consumption stream of each consumer? Is this allocation Pareto optimal? Is this market structure complete? (Hint: the equilibrium rate of interest and insurance premium are $r^* = 10$ and $q^* = \frac{1}{44}$, respectively.)

3. (30) Ann, Bob, and Carol are renting an apartment together and they must determine what level of ADSL service they will subscribe to. They can choose any nonnegative level of service $x \in \Re_+$, for which they will be charged $6x$ euros. Unfortunately, the three housemates do not have the same preferences for ADSL service. Each one's preference is described by a utility function of the form $u_i(x, y_i) = y_i + \alpha_i \log x$, where y_i denotes euros available to spend on other goods, and the values of α for Ann, Bob, and Carol are $\alpha_A = 4$, $\alpha_B = 8$, and $\alpha_C = 36$, respectively. Each of the housemates is endowed with 40 euros.

(a) Determine the level of ADSL service that they will subscribe to if it is determined by voluntary contribution.

(b) Identify all service levels that are consistent with Pareto optimality when $y_i > 0$, for $i = A, B, C$. Also calculate the set of Lindahl allocations and prices.

(c) Suppose the following procedure is used to decide upon the service level x on how much each will pay: Each housemate announces a service level $m \in \Re$; then they purchase a service level $x = m_A + m_B + m_C$, and each pays $\tau_A = (2 + m_B - m_C)x$, $\tau_B = (2 + m_C - m_A)x$, and $\tau_C = (2 + m_A - m_B)x$, respectively. Calculate the Nash equilibrium allocations for this procedure, and determine whether it is Pareto optimal.

4. (10) Study the existence, uniqueness and stability of competitive equilibria in a two-person two-good pure exchange economy where consumers are endowed with positive amounts of both goods, and their preferences over bundles $(x, y) \in \mathbb{R}^2_+$ are as follows: Consumer 1 lexicographically prefers good x to good y, and Consumer 2's marginal rate of substitution is a positive constant.

5. (10) Consider an economy where there is a public good (x), and a private good (y). The economy's initial endowment is $(0, \bar{y})$, but there is a technology that allows to produce public good using private good as input according to the production function $f(z) = \alpha z$. There are n individuals whose preferences over goods x and y are represented by continuously differentiable, strictly increasing and strictly quasi-concave utility functions, $u_1, ..., u_n$. Show that an allocation $(x, y_1, ..., y_n)$ is Pareto optimal for this economy if and only if it solves the problem

$$\max_{(x, y_1, ..., y_n, z) \in \mathbb{R}^{n+1}_+} u^1(x, y_1)$$

$$-u_i(x, y_i) \leq -\bar{u}_i, \text{ for } i = 2, ..., n,$$

$$x - f(z) \leq 0,$$

$$\sum_{i=1}^{n} y_i + z \leq \bar{y},$$

for some $(\bar{u}_2, ..., \bar{u}_n) \in \mathbb{R}$. Find the marginal conditions that characterize interior Pareto optimal allocations.