Exercise Set 3

1. There is only one perishable commodity available for consumption "today" and "tomorrow." Ms. A owns no good today, but wil1 own (for certain) 30 units of the good tomorrow. Her preference for alternative consumption streams is described by the utility function $u^A(x, y) = \min\{x, y\}$, where x and y denote her consumption today and tomorrow, respectively. Ms. B owns 20 units of the good today, and will own none tomorrow; her preference is described by the utility function $u^B(x, y) = xy$.

(a) If Ms. A and Ms. B engage in a borrowing-and-lending market (in which they are the only participants) in order to arrive at more desirable consumption streams than they are endowed with, and if each behaves "competitively" (taking the interest rate as given), what will be the equilibrium rate of interest and the equilibrium consumption stream of each?

(b) Verify that Walras's Law is satisfied at the equilibrium rate of interest. Is Walras's Law satisfied at any non-equilibrium interest rates, and if so, at which ones?

(c) Now suppose the womens' endowment streams are (15,15) for Ms. A and (5,15) for Ms. B. Is this a Pareto optimal allocation? Is there an interest rate at which this endowment stream is a Walrasian equilibrium?

For parts (d) and (e), assume that there is real investment process which yields three units of the good tomorrow for each unit of good today used as input. Also assume that the endowment streams are (20,0) for Ms. A and (10,0) for Ms. B, and that the investment process is operated by a competitive firm owned by Ms. A.

(d) Determine all Walrasian equilibrium interest rates, production plans, and consumption streams for each woman. (Hint: What is the firm's profit?)

(e) Consider the plan in which 15 units of the good are used today as input to the investment process, Ms. A's consumption stream is (5,5), and Ms. B's is (10,40). Find a Pareto improvement upon this plan (not necessarily a Pareto optimal plan) in which both women are made strictly better off.

2. Andy's income this year is \$20. He is retiring next year, and his income depends on whether Social Security remains financially sound (A), or it goes bankrupt and disappears (B). Andy's income in state A will be \$20, whereas it will be only \$10 (the amount paid by his private pension plan) in state B. Beth's income this year is also \$20, and her income next year depends on the state of nature: in state A Beth will have to contribute to SS, leaving her with an income of only \$10; in state B Beth's income is \$20. Andy's preference is represented by the utility function $u^1(x_0, x_A, x_B) = x_0 + 5 \ln x_A + 6x \ln_B$, and Beth's by the function $u^2(x_0, x_A, x_B) =$

$x_0+10\ln x_A+3\ln x_B$, where x_0 denotes the individual's spending this year, x_A denotes spending next year in state A, and x_B denotes spending next year in state B. Income is perishable; i.e., one unit of income dated this year is worthless next year.

(a) Determine the interior Pareto efficient allocation(s).

(b) Determine the Arrow-Debreu allocation and prices.

(c) Suppose that only spot markets and a credit market exist. Is the equilibrium allocation Pareto efficient? (Do not attempt to calculate the equilibrium.)

In (d), (e), and (f) you can solve directly, or you can appeal to the complete-markets security pricing formula.

(d) In the Arrow-Debreu market structure, what is the (implicit) interest rate?

(e) Suppose that there are only spot markets, and the only securities are shares in the firm Gamma Technologies and shares in the firm Delta Insurance. Each share of Gamma will yield \$2 in state A and \$1 in B. Each share of Delta will yield \$1 in state A and \$2 in B. Determine the equilibrium security prices and Andy's and Beth's holdings of securities.

(f) In the market structure in (e), what portfolio would one have to hold in order to guarantee oneself a return of \$1 next year whether the state is A or B? What would be the cost of that portfolio? What would you say is the interest rate?

3. There are two individuals in the economy, A and B, who care about their consumption today and tomorrow of a perishable good. The state of the world tomorrow, which will be revealed between today and tomorrow, can be either S (sunny) or C (cloudy). Both individuals are endowed with ten units of the good today and (wil1 be endowed for certain with) fifteen tomorrow, and their preferences are described by the (expected) utility function

$$U^i(x_0^i, x_S^i, x_C^i) = x_0^i + 5\pi^i \ln x_S^i + 5(1-\pi^i)\ln x_C^i,$$

where π^i denotes individual i's subjective probability assessment that state S will occur. Mr. A believes that the two states are equally likely, but Ms. B believes there is a $3/4$ chance the state will be S.

(a) Determine the interior allocations that are Pareto optimal.

(b) If the only market available is a borrowing and lending market (in which contracts are not state-contingent), what will the equilibrium interest rate (r) be, and how much will each individual lend (l)? (Normalize spot prices to one.)

(c) Assume that there are complete "Arrow-Debreu" contingent claims markets. Determine the equilibrium prices, consumptions, and the implicit interest rate. (Set $p_0 = 1$.)

(d) Now suppose that there is a borrowing and lending market as in b), and also a market for a real asset that pays off one unit of the good tomorrow if the state of the world is C and nothing if the state is S. Determine the equilibrium interest r and asset price p. How much will each individual save or borrow, and how many units of the asset will each one buy or sell?

4. Consider an economy with a single perishable consumption good (x), two periods (0 and 1), and two individuals (A and B). The probability that A (B) survives in period 1 is $\pi^A \in (0,1)$ ($\pi^B \in (0,1)$) independently of whether or not B (A) survives. Thus at $t = 1$ the are four states of nature $(s,t) \in \{A, \emptyset\} \times \{B, \emptyset\}$. Individuals preferences are described by the utility functions

$$U^A(x) = \alpha x_0 + \pi^A \pi^B \ln x_{AB} + \pi^A (1 - \pi^B) \ln x_{A\emptyset},$$

and

$$U^B(x) = \beta x_0 + \pi^A \pi^B \ln x_{AB} + \pi^B (1 - \pi^A) \ln x_{\emptyset B}.$$

Individual i's endowment, $i \in \{A, B\}$, is $\omega_0^i > 0$ at $t = 0$, and $\omega_1^i > 0$ if he survives at $t = 1$

(a) Determine the set of Pareto optimal allocations.

(b) Suppose that there is only one market for a real asset that pays one unit of the consumption good in state AB for p units of time-zero-consumption-good. Are the equilibrium allocations Pareto optimal.

(c) Calculate the Arrow-Debreu equilibria.