Exercise Set 2

1. Consider a pure exchange economy in which there are two goods (x and y) and 2 consumers whose preferences are represented by utility functions $u_i(x_i, y_i) = \alpha_i x_i + y_i$, where $\alpha_1 > \alpha_2$, and whose endowments are $(\bar{x}_i, \bar{y}_i) \gg 0$. Determine the set of Pareto efficient allocations. Is the competitive equilibrium an efficient allocation?

2. There are two persons (A and B), two goods (quantities are denoted by x and y), and no production is possible. A's MRS is 3 if $y > x$, and his MRS is $\frac{1}{2}$ if $y < x$. B's MRS is always 1. Draw an Edgeworth box diagram and indicate on the diagram the entire set of Pareto optimal allocations assuming there are 30 units of the x-good and 60 units of the y-good available to be distributed to A and B.

3. There are two persons (A and B), two goods (quantities are x and y), and no production is possible. Neither person is able to consume a negative quantity of either good, and the two people's preferences are described by the utility functions $u_A(x, y) = ax + y$ and $u_B(x, y) = x^b y$.

(a) Let w_x and w_y denote the available amounts of the two goods. Determine all the Pareto optimal allocations, expressing them in terms of a, b, w_x, and w_y. For each of the following three cases, draw an Edgeworth box diagram and indicate on the diagram the entire set of Pareto optimal allocations:

Case I: $\frac{w_x}{w_y} = \frac{b}{a}$; Case II: $\frac{w_x}{w_y} > \frac{b}{a}$; Case I: $\frac{w_x}{w_y} < \frac{b}{a}$.

(b) Let $a = b = 1$, let (0,5) be the bundle owned by Person A, and let (30,5) be the bundle owned by Person B. Determine the competitive (Walrasian) equilibrium price(s) and allocation(s).

4. There are two persons, Mr. A and Mr. B, and two goods, quantities of which will be denoted by x and y. Both persons have the same preference, described by the utility function,

$$u(x, y) = \begin{cases} x + y - 1 & \text{if } x \geq 1 \\ 3x + y - 3 & \text{if } x < 1 \end{cases}$$

There are 4 units of the x-good, all owned by Mr. A, and 6 units of the y-good, all owned by Mr. B.

(a) Determine all competitive equilibrium prices and allocations.

(b) In an Edgeworth box diagram, depict all Pareto optimal allocations.

5. Answer the following questions for the economy defined in 10 of Exercise Set 1.

(a) Consider the plan in which $z = 60$, $(x_{10}, y_{10}) = (24, 4)$; and $(x_i, y_i) = (4, 4)$ $i = 1, ..., 9$. Find a Pareto improvement upon this plan.

(b) Determine all the Pareto optimal production-and-consumption plans in which each consumer receives the same bundle as every other consumer.

6. In the economy defined in 11 of Exercise Set 1, is either of the institutional arrangements in (a) or (b) Pareto optimal for the residents of Dogpatch? If so, explain why; if not, find a Pareto improvement.

7. There are two persons (A and B) and two goods (wheat and bread), and one production process is available, a process by which one bushel of wheat can be turned into two loaves of bread. The individuals' preferences are described by the utility functions $u^A(x,y) = xy$ and $u^B(x,y) = x^2y$, where x is the person's consumption of wheat (in bushels) and y is the person's consumption of bread (in loaves). The two persons are endowed with a total of 60 bushels of wheat and no bread. For the consumption allocations in (a), (b), and (c) below, do the following: if the given allocation is Pareto optimal, then verify it; it the given allocation is not Pareto optimal, find a feasible Pareto improvement.

(a) $(x^A, y^A) = (12, 24)$ and $(x^B, y^B) = (24, 24)$.

(b) $(x^A, y^A) = (20, 20)$ and $(x^B, y^B) = (20, 20)$.

(c) $(x^A, y^A) = (20, 40)$ and $(x^B, y^B) = (10, 10)$.

8. There are three persons (A, B, and C), two goods (quantities are denoted by x and y), and no production is possible. Each person has the same preference, described by the utility function $u(x,y) = \sqrt{xy}$, where x and y denote the amounts allocated to him. A owns the bundle (12,4), B owns the bundle (4,4), and C owns the bundle (4,12). Determine whether each of the following allocations is in the core, and show why your answer is the right one.

(a) $(x_A, y_A) = (8, 8); (x_B, y_B) = (4, 4); (x_C, y_C) = (8, 8)$.

(b) $(x_A, y_A) = (9, 9); (x_B, y_B) = (4, 4); (x_C, y_C) = (7, 7)$.

9. Acca and Bayab are tiny islands in the Gulf of Ababa. Anna is the sole resident and owner of Acca, and Bob is the sole resident and owner of Bayab. Anna and Bob consume only apples and bananas, and each has the same preference, represented by the utility function $u(x,y) = xy$, where x and y represent the consumer's daily consumption of apples and bananas. Only apples are grown on Acca, on which the yield is 20 apples per day, and only bananas are grown on Bayab, where the yield is 10 bananas per day. Anna and Bob have been trading with one another (by boat) for years: every day Anna gives Bob 12 apples in return for'4 bananas. Call this situation and the resulting consumption allocation S, for "status quo".

(a) Is S Pareto optimal? Is it In the core? Verity your answers.

(b) Is S a "competitive" (Walrasian) allocation? If so, determine the associated competitive prices; if not, could both Anna and Bob be made better off by organizing their exchanges in terms of markets and prices?

Now suppose that Anna discovers a technology, called the Alpha technology, with which she can transform apples into bananas, at the rate of two apples for each banana obtained. Answer (c), (d), and (e) assuming that Anna is the sole owner of the Alpha technology.

(c) Is S Pareto optimal? If so, verify it; if not, find a Pareto improvement.

(d) How does the discovery of the new technology affect the set of core allocations? Is S in the core now?

(e) How does the discovery of the new technology affect the set of competitive allocations and prices?

(f) Now suppose it is not Anna who discovers a new technology, but Bob. Bob discovers the Beta technology, with which he can transform two bananas into one apple as often as he likes. Then is S in the core? Will the set of competitive allocations and/or prices be affected (as compared to the original, no-technology situation)?

(g) Now suppose that both technologies have been discovered. How will the set of competitive allocations and prices be affected by the ownership of the technologies. In particular, compare the competitive equilibria when Anna owns Alpha and Bob owns Beta with the equilibria when Anna owns Beta and Bob owns Alpha. What would happen to the competitive outcomes if one of the persons owned both technologies?

10. There are r girls and r boys, where r is a positive integer. The only two goods are bread and honey, quantities of which will be denoted by x and y (x denotes loaves of bread and y denotes pints of honey). Neither the girls nor the boys are well endowed: Each girl has 8 pints of honey but no bread, and each boy has 8 loaves of bread but no honey. Each girl's preference is described by the utility function $u^g(x, y) = \min\{ax, y\}$, and each boy's by the function $u^b(x, y) = x + y$.

(a) Determine the competitive excess demand function for honey and the competitive equilibrium prices and allocations.

(b) Determine the set of Pareto optimal allocations for $r = 1$ and for arbitrary r.

(c) Assume that $a = 1$. Determine the core allocations for $r = 1$, for $r = 2$, and for arbitrary r.

11. There are three consumers, each of whose preference is represented by the utility function $u(x, y) = xy$. The first consumer owns the bundle $(x, y) = (19, 1)$, the second owns the bundle (1,19), and the third owns the bundle (10,10). Determine the set of all core allocations.

12. Construct an example of an exchange economy to show that if the consumer types are not present in equal numbers, then it is not necessarily true that identical consumers are treated equally in core allocations.