Exercise Set 1

1. The excess demand for a particular good is given for $p > 0$ and $\alpha \in (0, \infty)$ by

$$E(p, \alpha) = D(p, \alpha) - S(p, \alpha) = 3 - (5 + \alpha) p + 5p^2 - p^3.$$

Determine the set of equilibria, and calculate $\frac{dp^*}{d\alpha}$.

2. Calculate and represent the offer curve of a consumer whose preferences are represented by $u(x, y) = y + \log x$, and whose endowments are $(2, 1)$.

3. There are two goods (x and y) and two consumers, Ann and Bob, who each owns three units of each good. Ann's MRS is $\frac{y}{x}$. Bob preferences are described as follows:

If $y < \frac{x}{2}$, then his indifference curve through (x, y) is horizontal

If $y > 2x$, then his indifference curve through (x, y) is vertical

If $\frac{x}{2} \leq y \leq 2x$, then his MRS is $\frac{x}{y}$.

Determine Bob's offer curve. (Note that it is not single valued at $p = 1$.) Also show that there is no market equilibrium. (Do this both by showing that Ann and Bob's offer curves "do not cross", and by showing analytically that the equilibrium condition cannot be satisfied.)

4. A small town produces a single product: apples. The town resources consist of two orchards, one containing only tall trees and the other only short trees, and two kinds of workers, giants and midgets. The technology for producing apples is such that one worker works with one tree to produce apples according to the following table:

	Tall Tree	Short Tree
Midget	1	3
Giant	8	4

There are 10 midgets and 20 giants in the town, and there are 40 tall trees and 5 short trees. None of the town's resources can be used for any other purpose.

(a) What is the efficient allocation of workers to trees? Are there any resources unemployed in this allocation? Determine the marginal product of each of the four resources in this allocation.

Trees owners pay workers a per apple wage. Each worker in the Tall Tree Orchard is paid w_T, and each worker in the Short Tree Orchard is paid w_S. Picked apples are sold by tree owners in the external apple market at price p.

(b) If workers are free to move between orchard, what conditions must w_T and w_S satisfy in order to sustain the efficient allocation?

(c) Under competitive conditions, what will be the equilibrium prices (w_T and w_S), and profits for the owners of resources? Determine whether in equilibrium any of the factor prices differ from the value of the factor's marginal product.

(d) Now suppose that it is the apple-pickers who sell the apples on the external market. Each worker hires a tree, paying the tree's owner for each apple the tree yields: r_T if the tree is tall, and r_S if it is a short tree. How will the competitive equilibrium differ from the one in (c)?

5. Let $I \subset \mathbb{R}$ be a bounded interval and let $f : I \to I$ be a continuous function.

(a) Use the Intermediate Value Theorem to show that f has a fixed point in I.

(b) Provide examples showing that if f is not continuous, I is not an interval, or I is not bounded, then f may not have a fixed point in I.

6. Two Manhattan pretzel vendors must decide where to locate their carts along a given block of Sixth Avenue. Represent each possible location by a point in the unit interval, $I = [0, 1]$. The profit of vendor i depends continuously on both vendors' locations; i.e., for $i \in \{1, 2\}$, $\pi_i : I^2 \to \Re$ is continuous. Moreover, each π_i is a quasi-concave function; i.e., $U^i(\bar{x}) = \{x \in I^2 \mid \pi_i(x) \geq \pi_i(\bar{x})\}$ is a convex set for each $\bar{x} \in \Re$. An equilibrium is a profile of locations (x_1, x_2) such that $\pi_1(x_1, x_2) \geq \pi_1(x_1', x_2)$ for each $x_1' \in I$, and $\pi_2(x_1, x_2) \geq \pi_2(x_1, x_2')$ for each $x_2' \in I$. Prove that an equilibrium exists.

7. There are two persons (A and B), two goods (quantities are denoted by x and y), and no production is possible. A owns the bundle (20,60) and her MRS is 3 if $y > x$, and $\frac{1}{2}$ if $y < x$. B owns the bundle (20,60) and his MRS is always 1. Determine the competitive (Walrasian) equilibrium price(s) and allocation(s).

8. Consider a pure exchange economy in which there are only two goods (x and y) and only two persons, Ann and Bob. Ann owns eight units of the x-good and none of the y-good and her preferences are described by the utility functions $u_A(x_A, y_A) = x_A y_A$. Bob owns three units of the y-good and none of the x-good, and his preferences are described by the utility function $u_B(x_B, y_B) = y_B + \ln x_B$. Determine the set of competitive equilibrium prices and allocations.

9. Consider a pure exchange economy in which there are two goods (x and y) and n consumers whose preferences are represented by utility functions $u_i(x, y) = \alpha_i x + y$, where $0 < \alpha_1 < \alpha_2 < ... < \alpha_n, .$ and whose endowments are $(\bar{x}_i, \bar{y}_i) \gg 0$.

(a) Prove that this economy has a competitive equilibrium.

(b) Determine whether there is a unique equilibrium price ratio and/or allocation.

10. There are only two goods, grapes and wine. There is a single production process (or "firm") available, which transforms grapes into wine according to the production

function

$$f(z) = \begin{cases} 0 & \text{if } 0 \leq z \leq 20 \\ z - 20 & \text{if } 20 < z \leq 80 \\ 20 + \frac{1}{2}z & \text{if } z > 80, \end{cases}$$

in which z denotes the pounds of grapes used as input and $f(z)$ the resulting quarts of wine obtained as output. There are ten identical consumers, each with a preference described by the utility function $u(x, y) = xy$, where x and y denote, respectively, pounds of grapes and quarts of wine consumed. Each consumer owns 12 pounds of grapes and no wine at all, and has $\frac{1}{10}$ of the firm's property. The firm and the consumers all behave "competitively" (i.e., they are price-takers).

(a) Draw the set of all the aggregate consumption bundles (x, y) that are feasible.

(b) Write down the firm's demand correspondence for grapes, being careful to indicate the price lists (p_x, p_y) for which the firm's demand is not defined.

(c) Assume that the price of grapes is three dollars per pound and the price of wine is five dollars per quart. How much will each consumer demand of each good?

(d) Verify that there is no competitive equilibrium.

11. The tiny country of Dogpatch has only 90 residents and ten identical machines. Ten of the people (called "capitalists") own one machine apiece but are unable to provide any useful labor services. Each of the remaining 80 people (called "workers") has the capacity to work with machines and other workers to produce shmoos, but none of the workers owns any machines. Combining x workers with y machines yields $F(x, y) = x^{\alpha}y^{1-\alpha}$ shmoos, where $\alpha = \frac{2}{3}$. Note that it is possible to divide a worker's time among any number of machines and to divide a machine's time among any number of workers. No one in Dogpatch cares about consuming shmoos, but shmoos can be sold in the neighboring country of Alcappia for a dollar apiece. Everyone in Dogpatch uses dollars to purchase in Alcappia the goods that he does care about consuming. All residents of Dogpatch are price-takers in all markets, and everyone understands how to use the constant-returns-to-scale technology embodied in the function F. Machines and labor are neither imported nor exported by Dogpatch.

(a) Suppose that the capitalists are the entrepreneurs: each one hires workers and combines them with his machine, and then sells the resulting production of shmoos. What will be the equilibrium wage rate and the total production of shmoos, and how many dollars will each capitalist and each worker spend in Alcappia?

(b) Now suppose that the workers are the entrepreneurs: each worker rents machine time, which he combines with his own labor, and then sells the resulting production of shmoos. What will be the equilibrium rental price and the total production of shmoos, and how many dollars will each capitalist and each worker spend in Alcappia?