Stability of Competitive Equilibrium

I. **Definitions**.

1. A *price adjustment rule* is a function $f : S^l \to S^l$ such that $f_k(p) > p_k$ whenever $z_k(p) > 0$, and $f_k(p) < p_k$ whenever $z_k(p) < 0$, for $k \in \{1, \dots l\}$.

2. An equilibrium price vector p^* is *locally stable* if there are a price adjustment rule f and a positive number ε such that for any $p \in B_\varepsilon(p^*)$ the sequence $\{p(n)\}_{n=1}^{\infty}$, where $p(1) = p$ and $p(n) = f(p(n-1))$, converges to p^*.

3. It is *globally stable* if there is a price adjustment rule f such that for any $p \in S^l$ the sequence $\{p(n)\}_{n=1}^{\infty}$, where $p(1) = p$ and $p(n) = f(p(n-1))$, converges to p^*.

II. **Exercise**. (Due to D. Gale).

There are two persons, two goods, and no production is possible. Consumer 1 owns the bundle (1,0) and her preferences are described by the utility function $u_1(x, y) = \min\{x, 2y\}$. Consumer 2 owns the bundle (0,1) and her preferences are described by the utility function $u_2(x, y) = \min\{2x, y\}$. Calculate the set of competitive equilibrium price vectors in $S = \{(p, 1-p),\ p \in [0, 1]\}$, and determine which of the equilibria are locally and/or globally stable.