**Final Exam** (April 2011)

1. (40 points) Ann and Bob are the only inhabitants of an island where the only food available is fish ($x$) and fruit ($y$). Their dietary preferences are described by the utility functions $u^A(x,y) = 2\ln x + y$ and $u^B(x,y) = x^2 y$, respectively. Ann is very able climbing trees and can collect as much as 8 pounds of fruit a day, but she cannot swim, and is therefore unable to fish. Bob is an able fisherman whose daily catch is 12 pounds of fish; however, he suffers from dizziness (vertigo) and is unable to climb up a tree to collect fruit.

(a) Calculate the set of Pareto optimal allocations, the core, and the set of competitive equilibrium allocations of the economy.

(b) Carol and David, two survivors of a shipwreck, arrive at the island. Their production abilities and preferences are identical to those of Ann and Bob, respectively. Calculate the competitive equilibrium of the economy formed by Ann, Bob, Carol and David. Show that the set of competitive equilibria is invariant to replicating the economy.

(c) Are the allocations $[(12,8),(0,0),(12,8),(0,0)]$ and $[(4,6),(8,2),(12,8),(0,0)]$ in core the economy formed by Ann, Bob, Carol, and David? Show that the allocations in the core of this economy satisfy $(x_A, y_A) = (x_C, y_C)$ and $(x_B, y_B) = (x_D, y_D)$.

2. (20 points) Consider an economy that extends over two periods, today and tomorrow, and where there are two consumers, $A$ and $B$, and a single perishable good. The state of nature tomorrow can either be $H$ or $L$. Consumers' preferences over consumption today and tomorrow are represented by the utility functions $u^A(c_0, c_H, c_L) = c_0 c_H$, and $u^B(c_0, c_H, c_L) = c_0 c_L$. Both $A$ and $B$ are endowed with four units of the good in each of the two periods (regardless of whether the state in the second period is $H$ or $L$).

(a) Determine the Arrow-Debreu competitive equilibrium prices and allocations.

(b) Suppose that the only market available is a credit market. What will be the competitive equilibrium interest rate $r$ and how much will each person borrow or lend? Is the resulting allocation Pareto optimal?

(c) Suppose that in addition to a credit market there is also a security that pays one unit of consumption tomorrow only if event $H$ occurs. Let $q$ denote the market price of this security (in units of consumption today). Determine the competitive equilibrium interest rate and security price. How much will each person borrow or lend? How much will each person buy or sell of the security? Does equilibrium involve either $A$ or $B$ *selling short*? (Hint: use the consumers' budget constraints to identify the equilibrium interest rate and security price.)

3. (20 points) A small town must decide the number of hours of street cleaning service $x$ it will have. The cost of street cleaning services is $c > 0$ euros/hour. Each citizen $i \in \{1, ..., n\}$ is endowed with $\bar{y}_i$ euros, and her preferences are described by a utility function of the form $u^i(x,y) = y + a_i v(x)$, where $y$ denotes income (in euros) available to spend on other goods, $v$ is an increasing, continuously differentiable and concave function, and $a_i > 0$ measures citizen $i$'s *intensity* of preference for street cleaning service.

(a) Identify the conditions that characterize interior Pareto optimal, and Lindahl equilibrium allocations for this economy.

(b) Show that if the number of hours of street cleaning service is determined by voluntary contribution, then the resulting allocation is suboptimal.

(c) Apply your findings in (a) and (b) to the case $c = 6$ and $v(x) = 2\sqrt{x}$, assuming there are only three individuals whose intensity parameters are $a_1 = 2$, $a_2 = 4$, and $a_3 = 6$.

(d) For the economy described in (c), determine the allocations induced by the mechanism $(M, g)$ defined by $M_i = \mathbb{R}$, and for $m \in \mathbb{R}^3$ $g(m) = (x, y_1, y_2, y_3)$, where $x = \sum_{i=1}^{3} m_i$, $y_1 = \bar{y}_1 - (2 + m_2 - m_3)\, x$, $y_2 = \bar{y}_2 - (2 + m_3 - m_1)\, x$, and $y_3 = \bar{y}_3 - (2 + m_1 - m_2)\, x$ . (You must identify the Nash equilibria of the game induced by $(M, g)$.)

4. (20 points) Consider the following informal claims about a 2-person, 2-good pure exchange economy:

(2.1) The economy has a competitive equilibrium.

(2.2) Every Pareto optimal allocation can be supported as a competitive equilibrium.

(2.3) Every competitive allocation is in the core of the economy.

For one these claims (anyone):

(i) establish a set of conditions under which it holds, and

(ii) provide a proof. (Make sure to identify the role of each condition in the proof.)