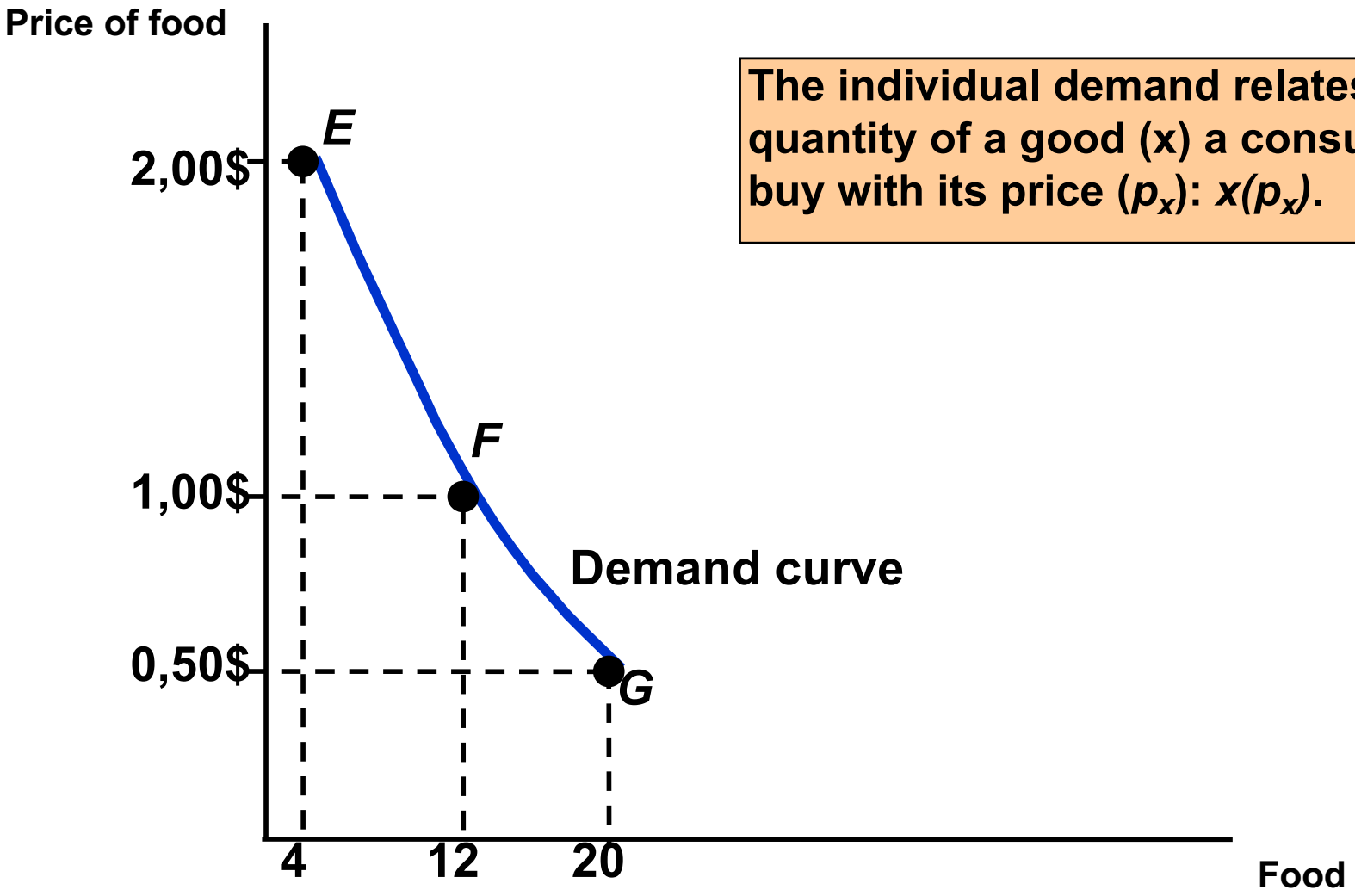
Consumer theory

Demand Functions

Substitution and Income Effects

The Demand Curve



The individual demand relates the quantity of a good (x) a consumer will buy with its price (p_x): $x(p_x)$.

Example: Cobb Douglas utility function

Data: $u(x,y) = x^{1/2} y^{1/2}$ $\quad p_y = 2 \quad I = 80$

Calculate the demand function for good x

Solution: We solve the system:

(a) $MRS = p_x / p_y$

(b) $p_x x + p_y y = I$

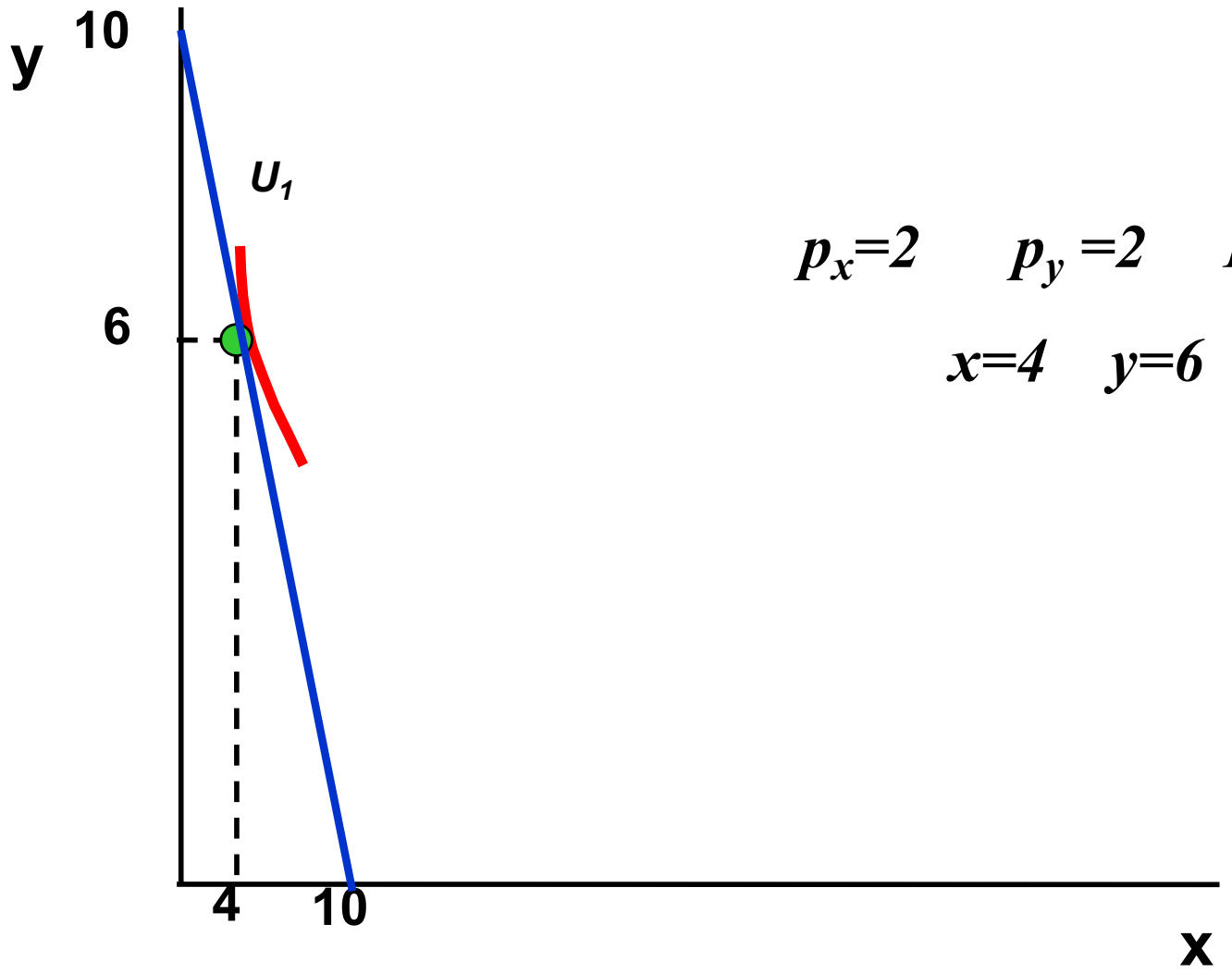
In our case,

(a) $y/x = p_x/2 \Rightarrow 2y = p_x x$

(b) $p_x x + p_y y = I \Rightarrow p_x x + 2y = 80 \Rightarrow 2p_x x = 80$

$\Rightarrow x^d(p_x) = 40/p_x$

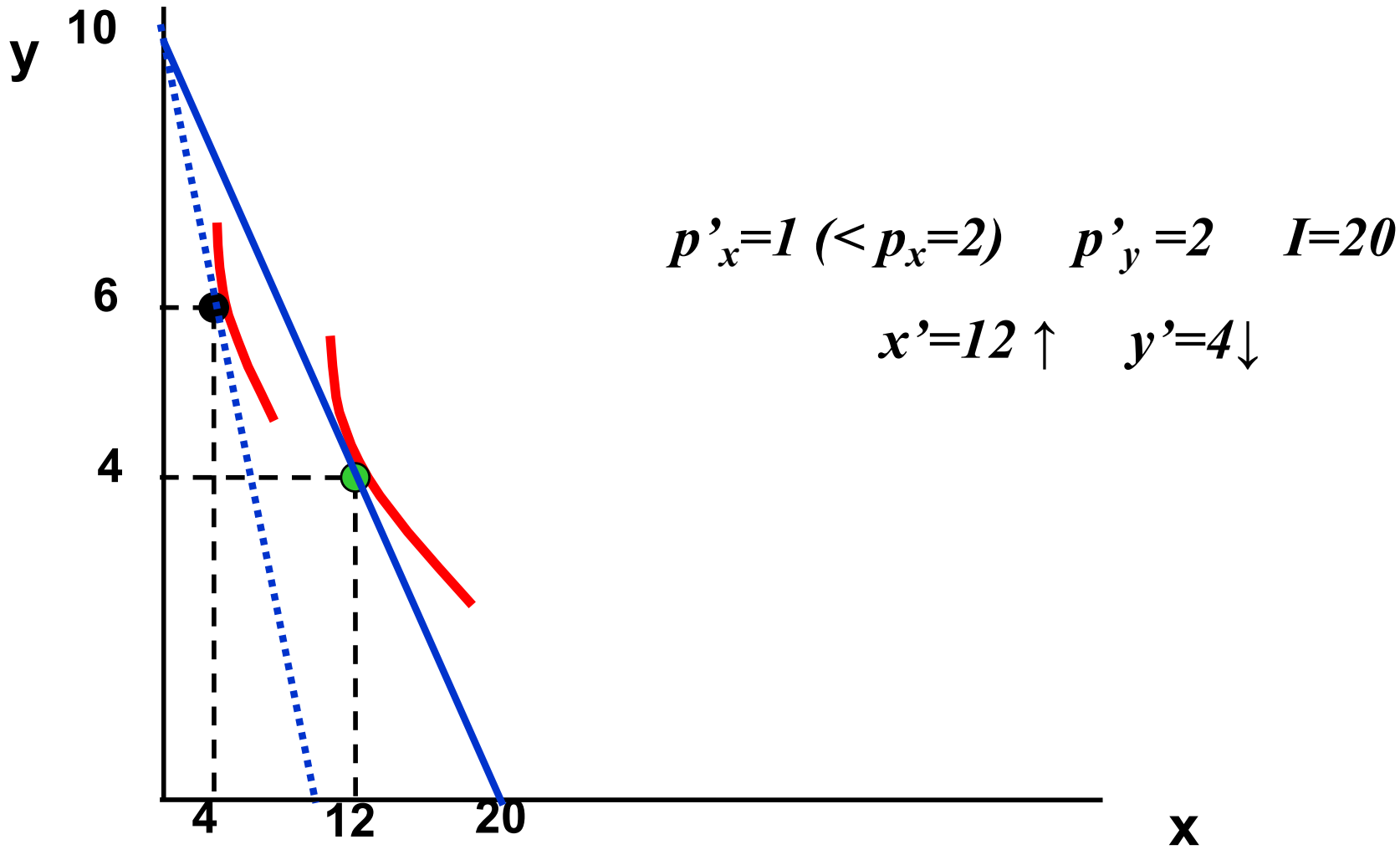
Effect of a variation in the price of *x*



$p_x=2 \quad p_y=2 \quad I=20$

$x=4 \quad y=6$

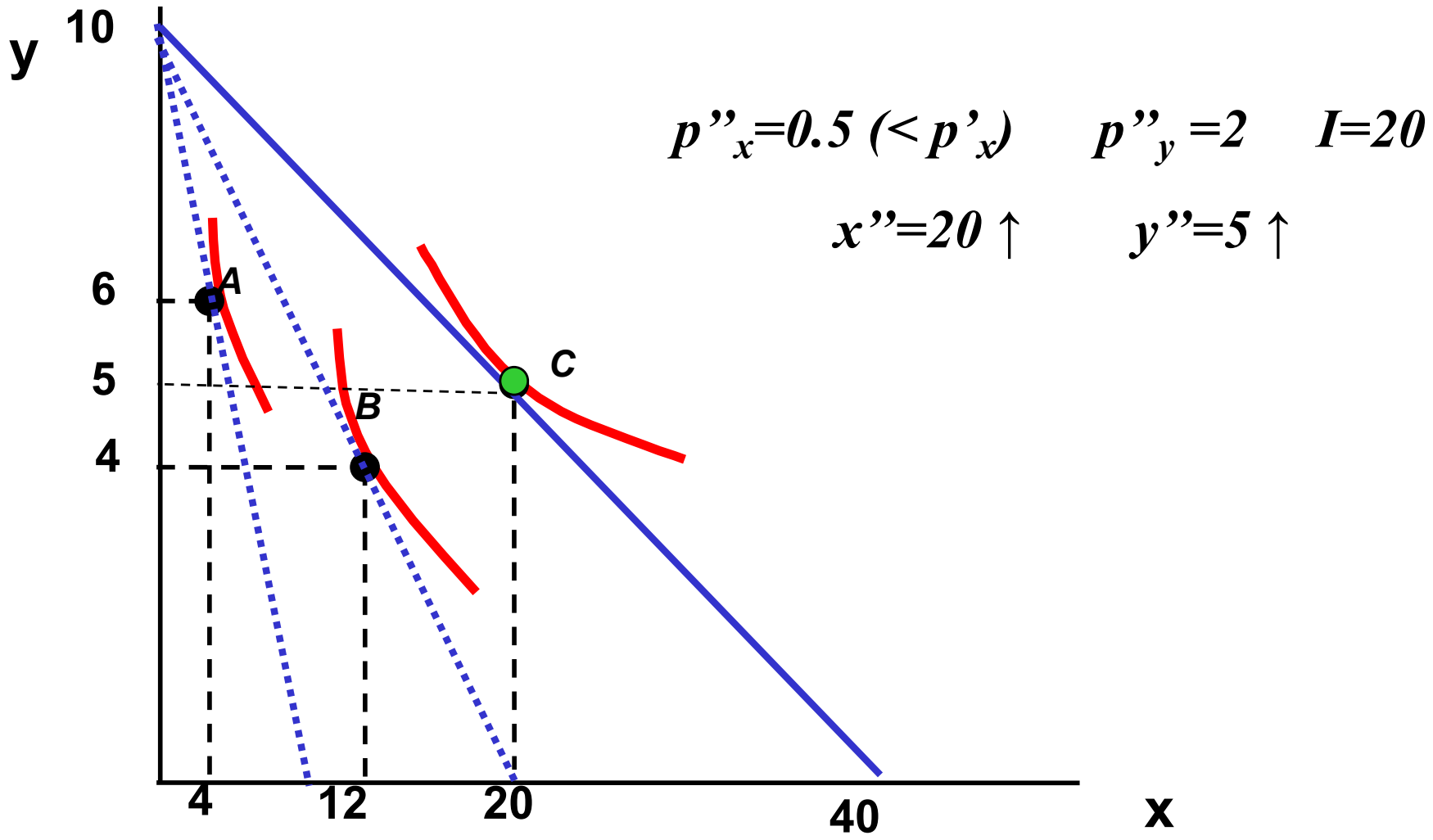
Effect of a variation in the price of *x*



$p'_x=1\ (<p_x=2)$ $p'_y=2$ $I=20$

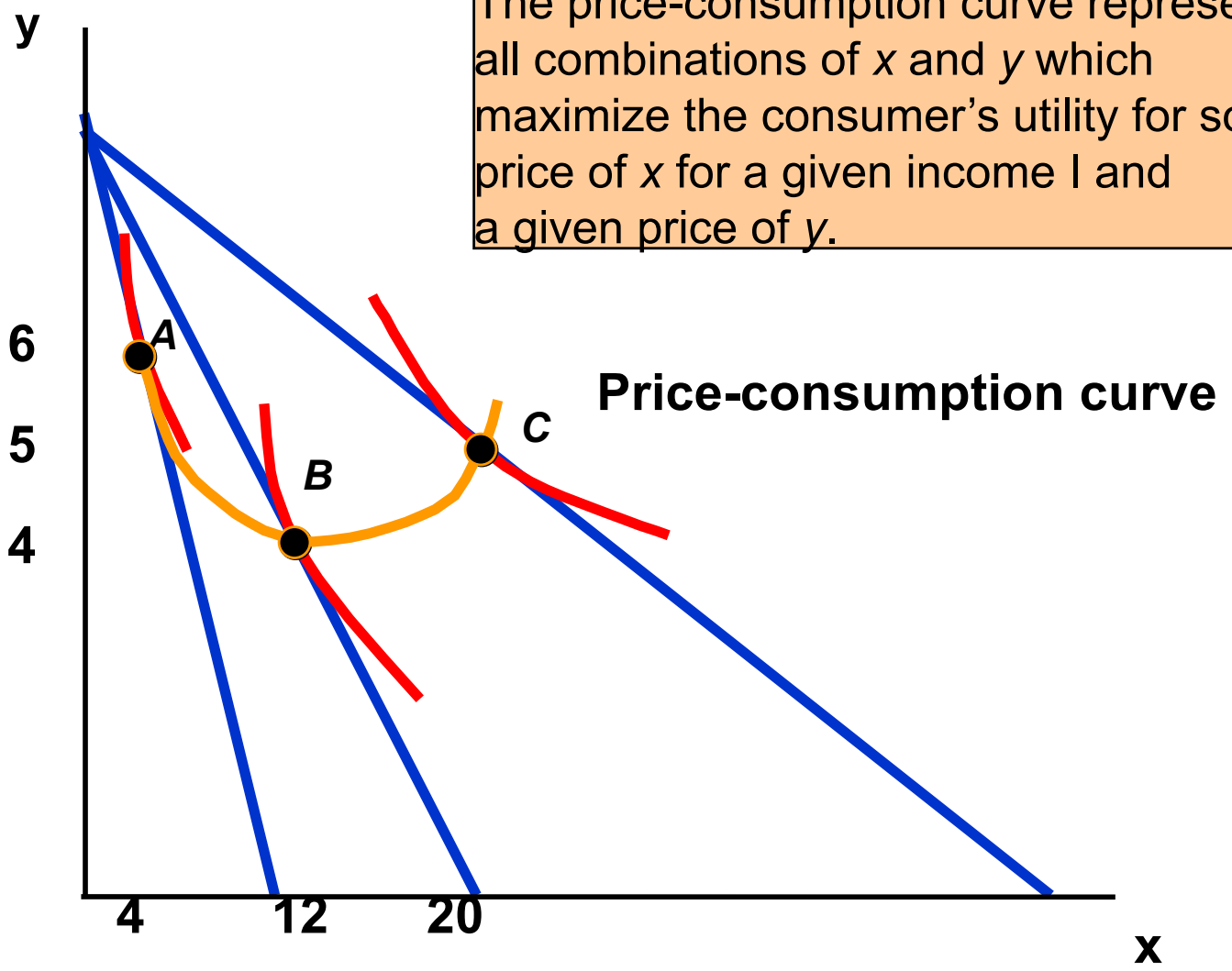
$x'=12\uparrow$ $y'=4\downarrow$

Effect of a variation in the price of *x*

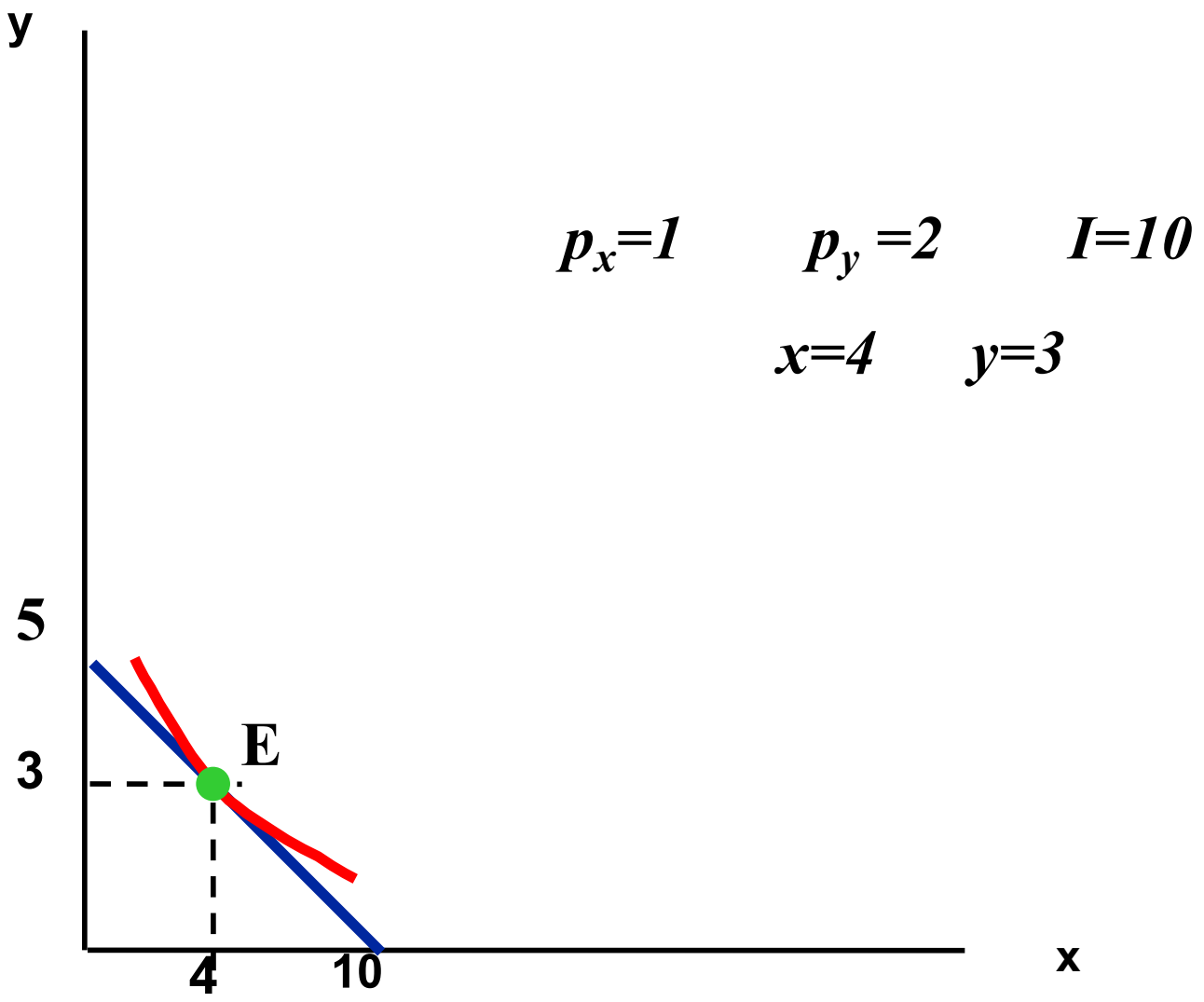


The price-consumption curve

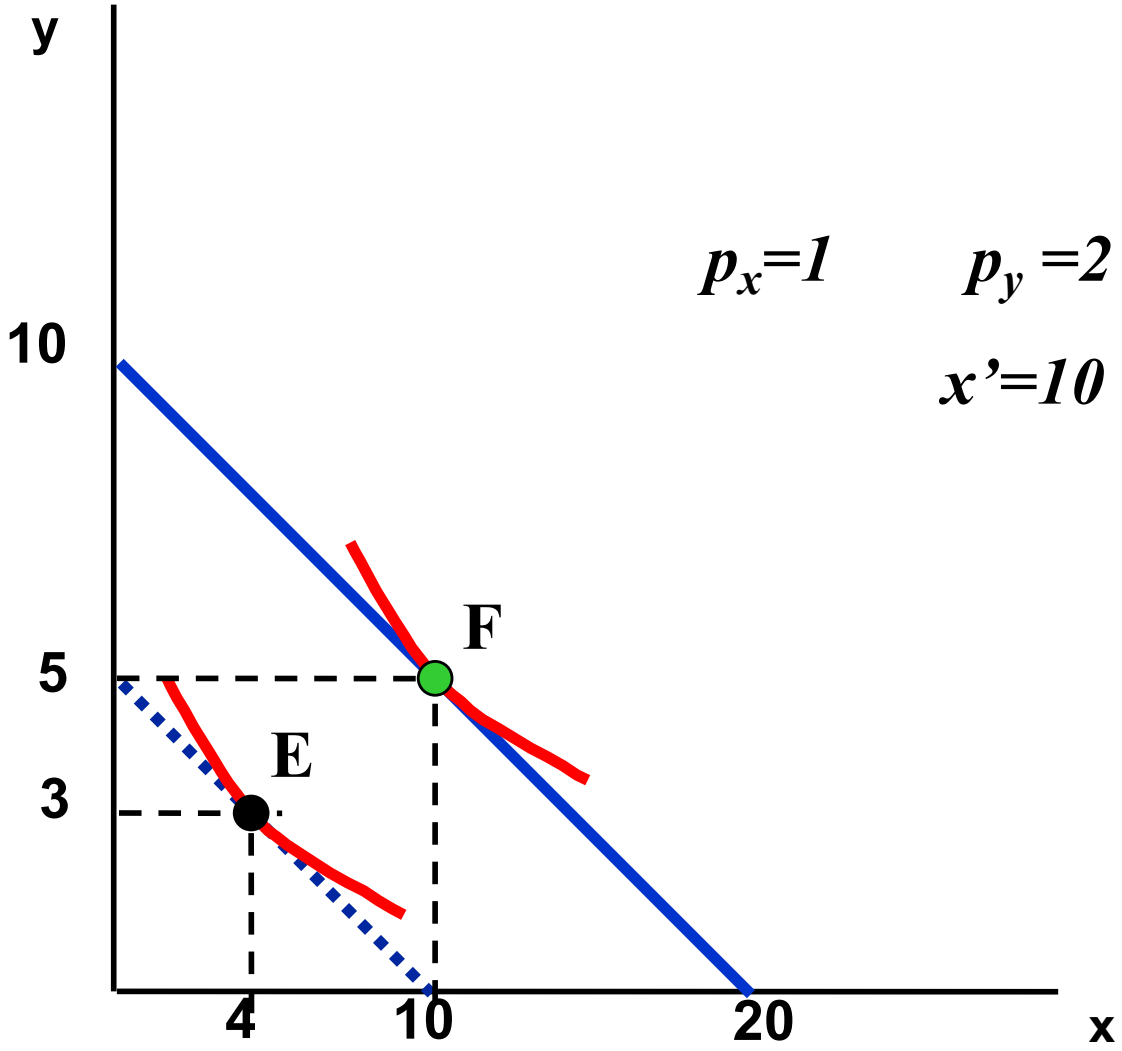
The price-consumption curve represents all combinations of *x* and *y* which maximize the consumer's utility for some price of *x* for a given income I and a given price of *y*.



Effects of a variation in income



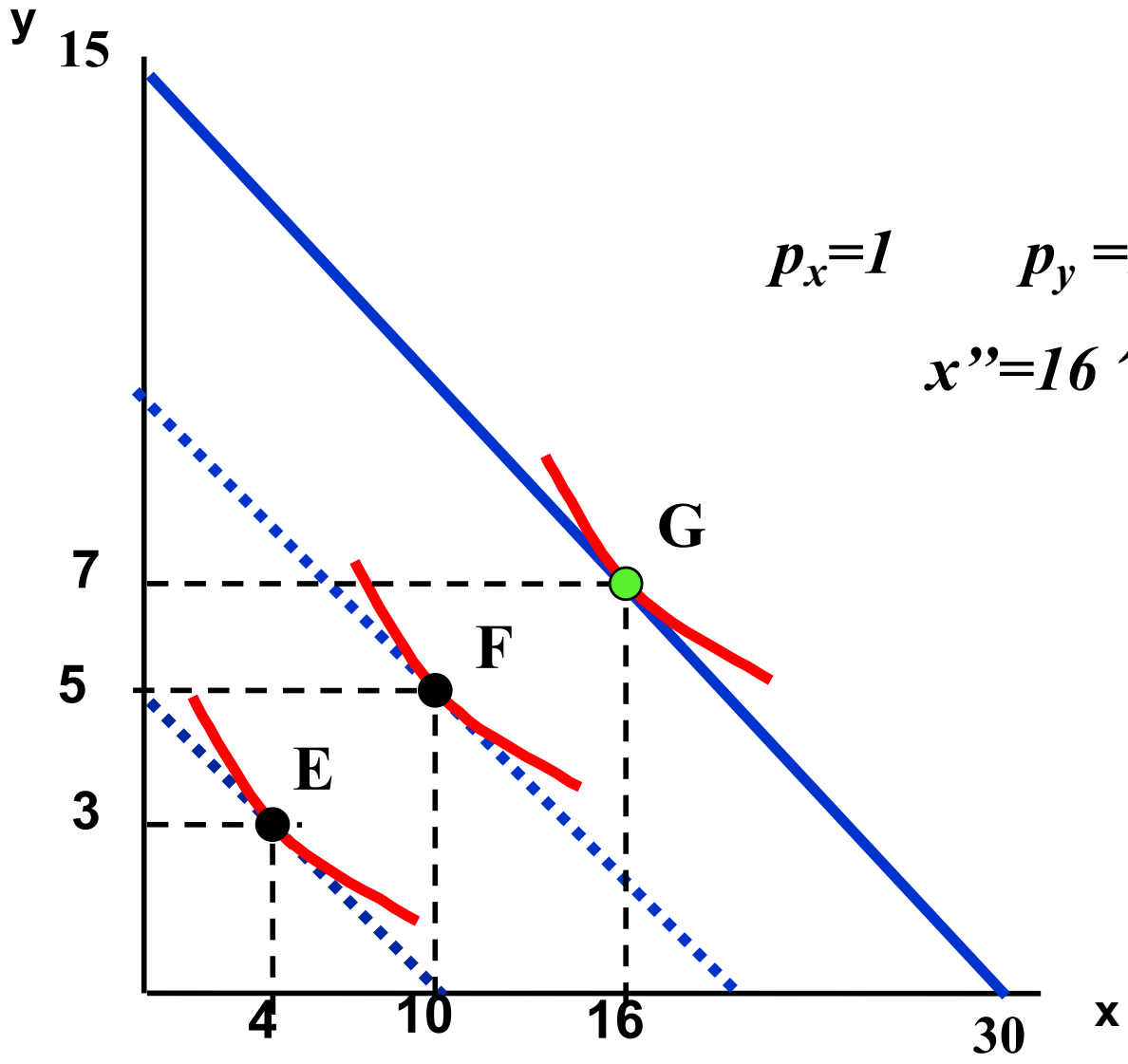
Effects of a variation in income



$p_x=1$ $p_y=2$ $I'=20\ (>I)$

$x'=10\uparrow$ $y'=5\uparrow$

Effects of a variation in income

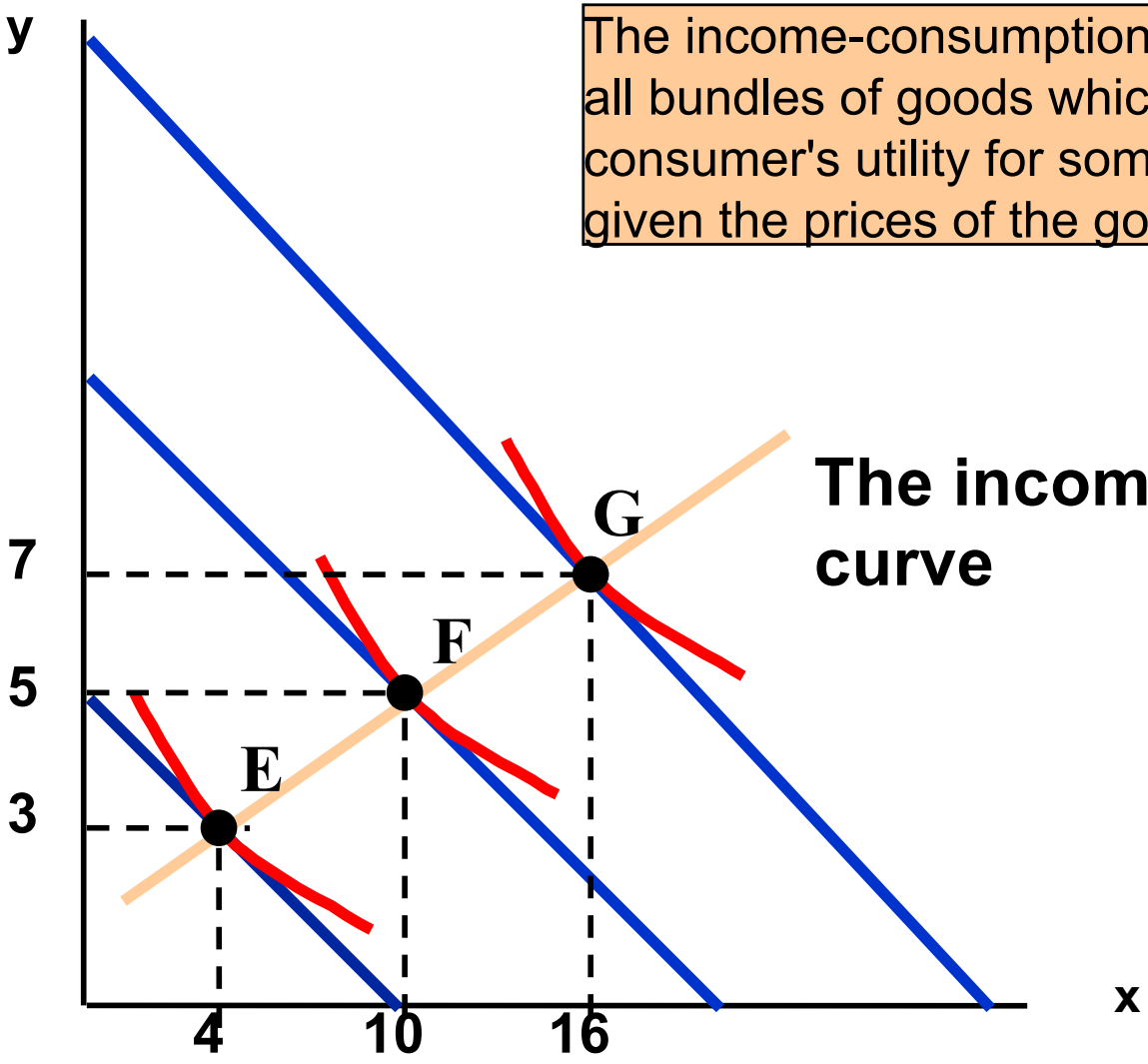


$p_x=1$ $p_y=2$ $I''=30\ (>I')$

$x''=16\uparrow$ $y''=7\uparrow$

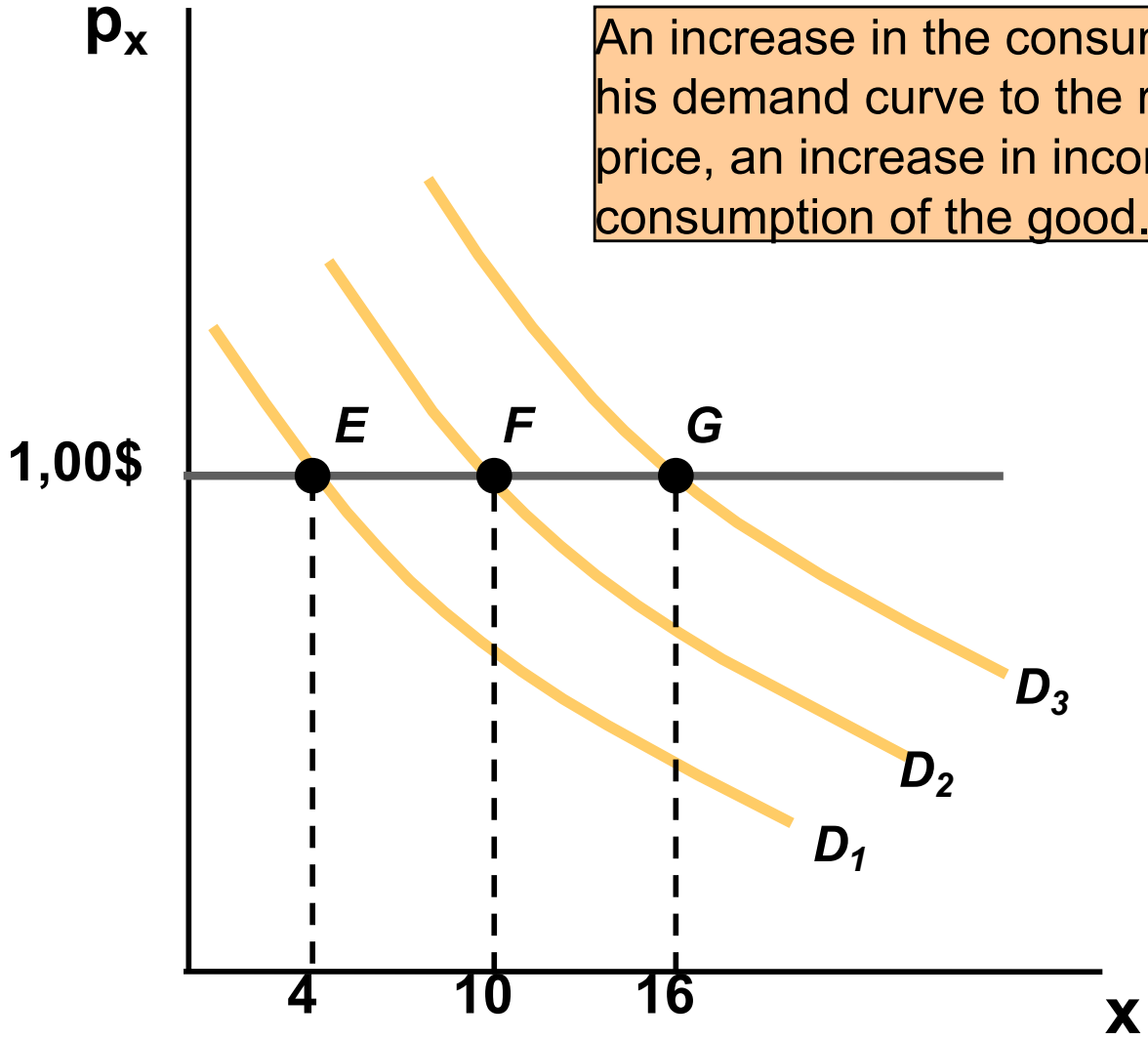
The income-consumption curve

The income-consumption curve represents all bundles of goods which maximize the consumer's utility for some level of income, given the prices of the goods.



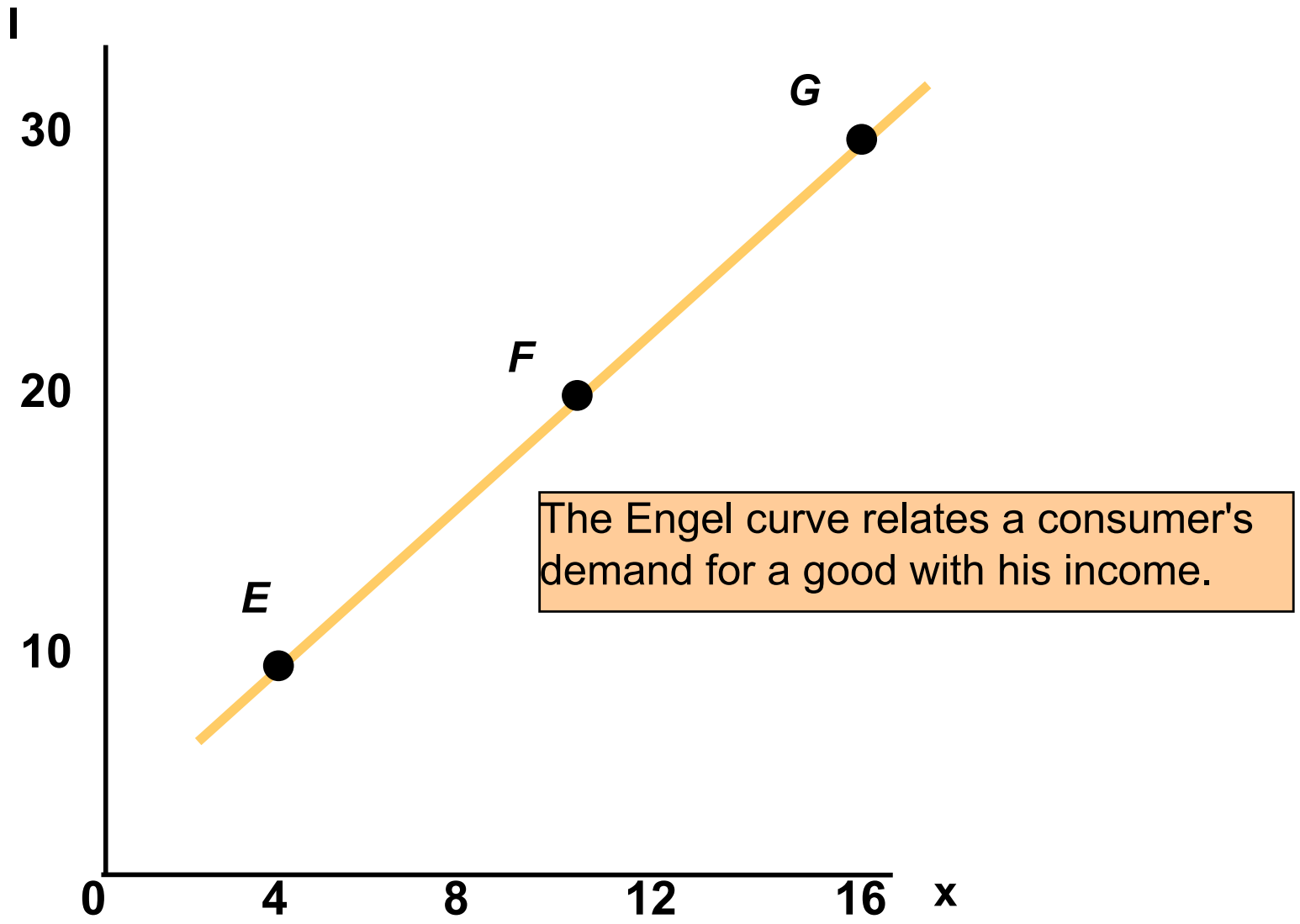
The income-consumption curve

Effects of a variation in income



An increase in the consumer's income shifts his demand curve to the right: for the same price, an increase in income leads to a higher consumption of the good.

The Engel curve



Example: Cobb-Douglas utility function

Data: $u(x,y) = x^{1/2} y^{1/2}$ $\quad p_x = 1 \quad p_y = 2$

Calculate the Engel curve for good x

Solution: We solve the system:

(a) $MRS = p_x / p_y$

(b) $p_x x + p_y y = I$

In our case,

(a) $y/x = 1/2 \Rightarrow 2y = x$

(b) $p_x x + p_y y = I \Rightarrow x + 2y = I \Rightarrow 2x = I$

$\Rightarrow x^E(I) = I/2$

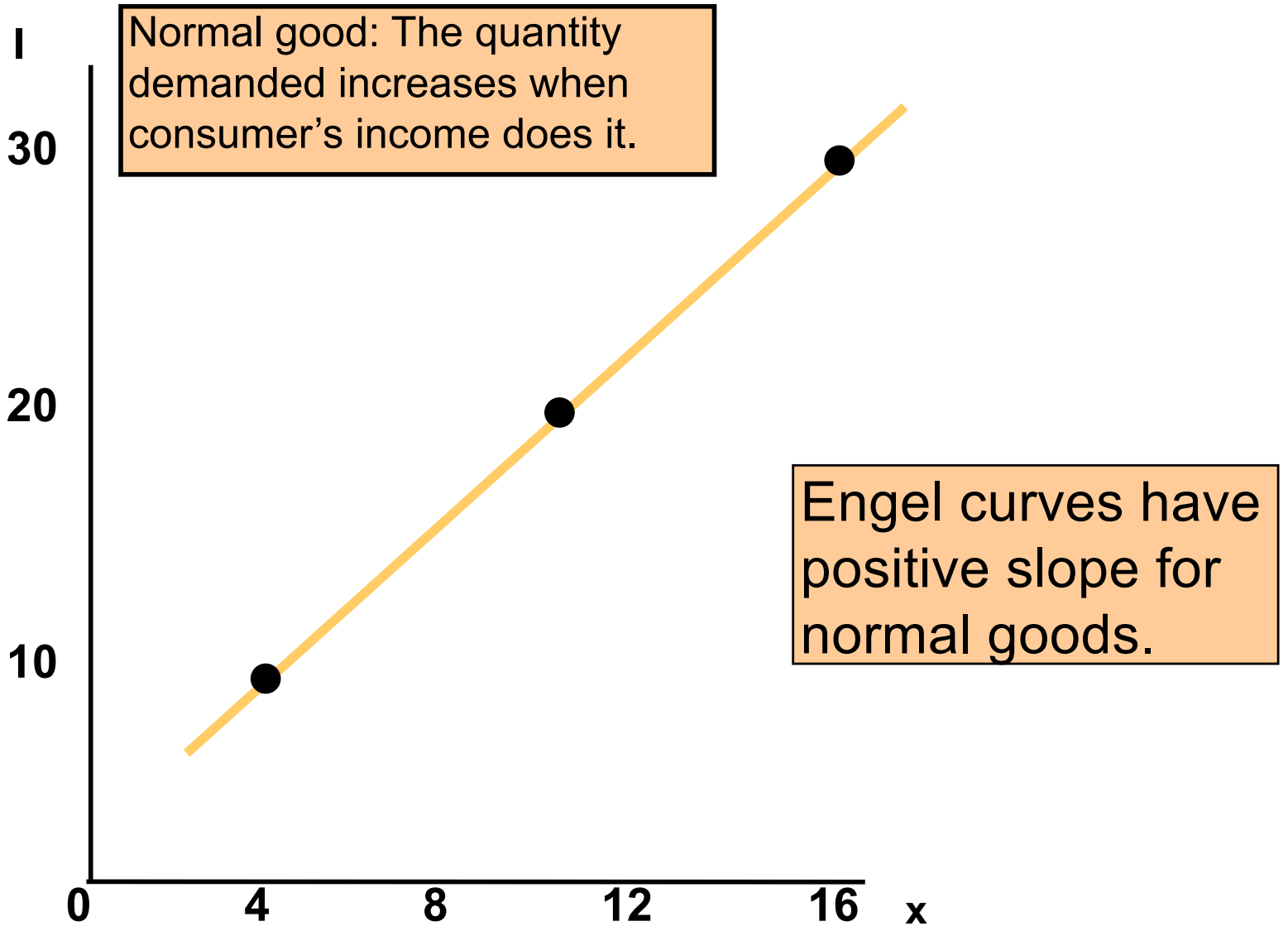
The individual demand

Goods can be classified as normal or inferior, defined as:

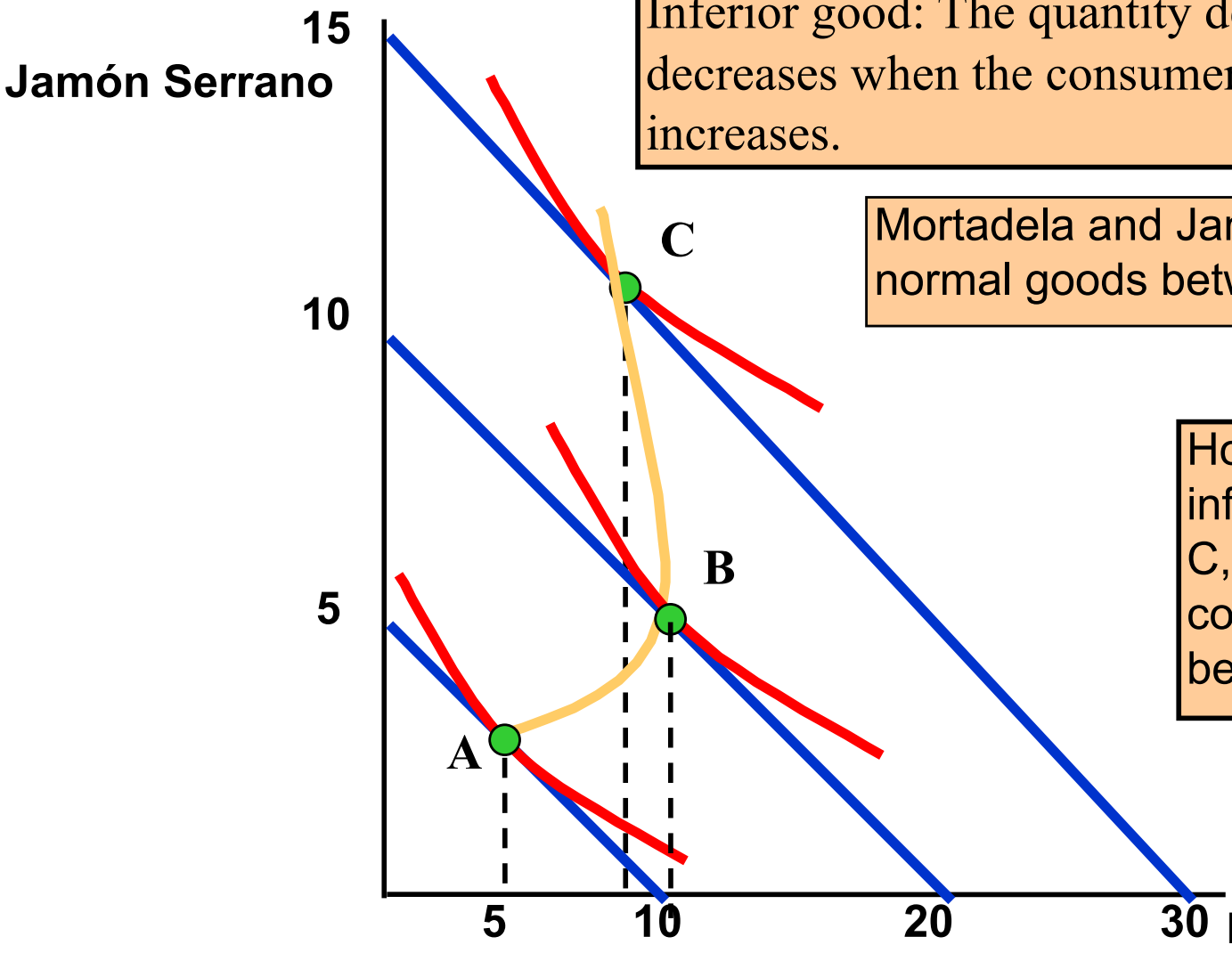
Normal goods: The quantity demanded increase with the income (therefore, the Engel curve has a positive slope).

Inferior goods: The quantity demanded of the good decreases with the income (therefore, the Engel curve has a negative slope).

The Engel curve for normal goods



Inferior goods

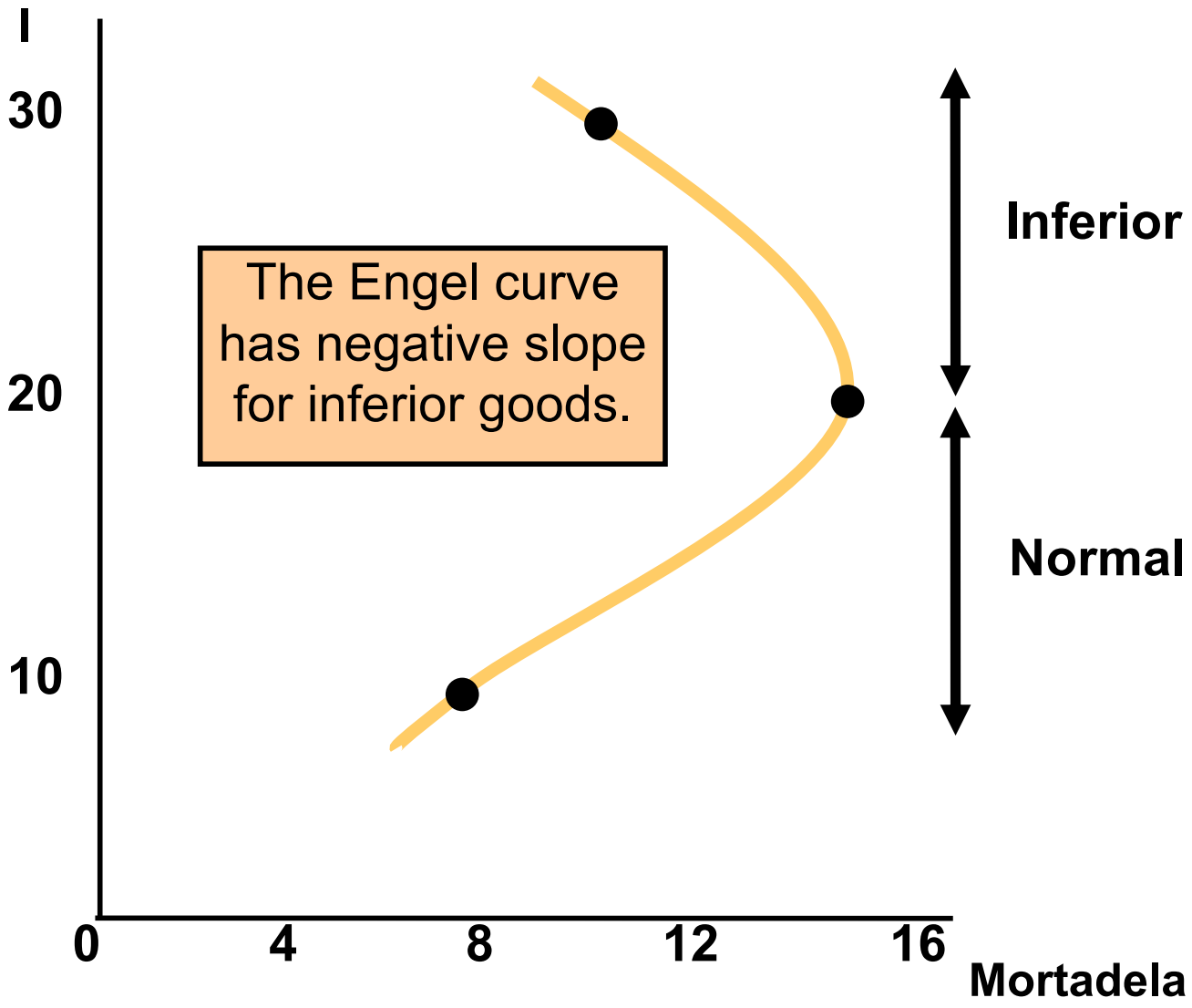


Inferior good: The quantity demanded decreases when the consumer's income increases.

Mortadela and Jamón are both normal goods between A and B.

However, mortadela is inferior between B and C, where the income-consumption curve bends backwards.

Engel curve for inferior goods



Consumption expenditure
Encuesta de Presupuestos Familiares (Spain)

Estructura de gasto por quintil y grupos de gasto. Año 2022

Grupos de gasto	Quintiles					
	Total	Quintil 1	Quintil 2	Quintil 3	Quintil 4	Quintil 5
TOTAL	100,0	100,0	100,0	100,0	100,0	100,0
1. Alimentos y bebidas no alcohólicas	16,0	20,5	19,5	18,1	15,7	12,4
2. Bebidas alcohólicas y tabaco	1,5	1,5	1,6	1,6	1,7	1,4
3. Vestido y calzado	3,9	3,2	3,9	3,9	4,0	4,0
4. Vivienda, agua, electricidad, gas y otros combustibles	32,4	42,6	37,8	34,2	31,7	27,0
5. Muebles, artículos del hogar y artículos para el mantenimiento corriente del hogar	4,1	3,0	3,3	3,8	4,2	4,8
6. Sanidad	3,9	2,6	3,3	3,8	4,0	4,5
7. Transporte	12,0	6,6	8,0	9,6	11,6	16,7
8. Comunicaciones	2,9	4,4	3,8	3,2	2,7	2,1
9. Ocio y cultura	4,9	2,7	3,6	4,3	5,0	6,1
10. Enseñanza	1,5	0,9	1,1	1,5	1,6	1,7
11. Restaurantes y hoteles	9,4	4,9	6,8	8,5	10,0	11,7
12. Otros bienes y servicios	7,5	7,1	7,4	7,4	7,7	7,5

Individual demand

Two goods x and y are gross substitutes if the increase (decrease) of the price of one of the goods leads to an increase (decrease) of the quantity demanded of the other good.

Example: cinema tickets and video rentals.

Individual demand

Two goods x and y are gross complements if an increase (decrease) in the price of one good results in a decrease (increase) of the quantity demanded of the other good.

Example: petrol and cars

Individual demand

Two goods x and y are independent if a variation in the price of one good does NOT affect to the quantity demanded of the other good.

Example: cinema tickets and milk.

Example: Cobb-Douglas utility function

Data: $u(x,y) = x^{1/2} y^{1/2}$

Calculate the general formula of the demand functions for goods x and y as a function of prices and income.

Solution: We solve the system composed of

(a) $MRS = p_x / p_y$

(b) $p_x x + p_y y = I$

In our case,

(a) $y/x = p_x/p_y \Rightarrow p_y y = p_x x$

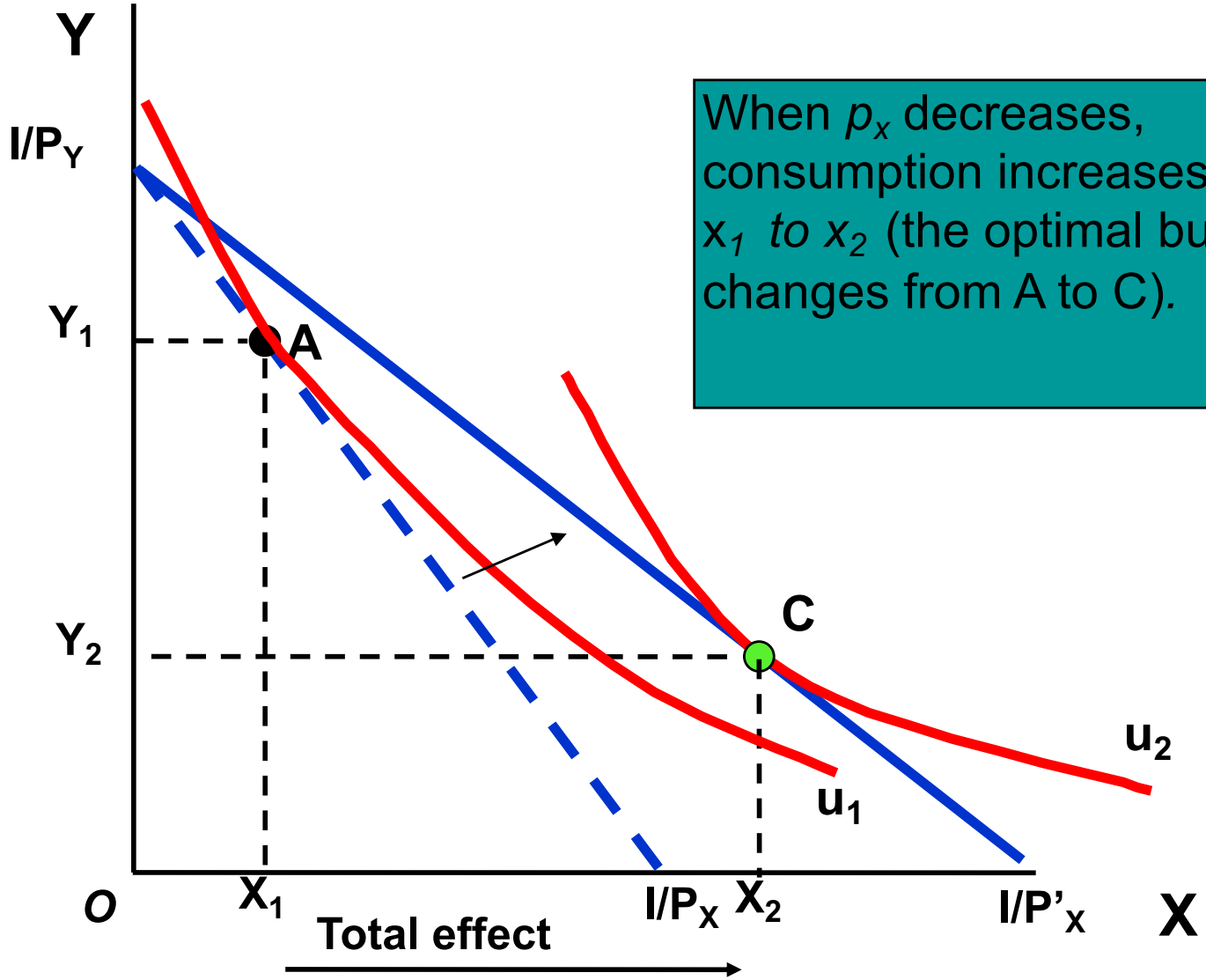
(b) $p_x x + p_y y = I \Rightarrow 2p_x x = I \Rightarrow x^d(p_x, p_y, I) = I/(2p_x)$

$$y^d(p_x, p_y, I) = I/(2p_y)$$

x and y are normal and independent goods.

Substitution and Income Effects

Let's see graphically the effect of $\downarrow p_x$



When p_x decreases, consumption increases from x_1 *to* x_2 (the optimal bundle changes from A to C).

Substitution and income effects

The reduction of the price of a good has two effects over consumption:

(1) Consumers buy a higher quantity of the good because, now, it is cheaper and, therefore, the other goods are relatively more expensive. This effect caused by the variation of the relative prices is called SUBSTITUTION EFFECT.

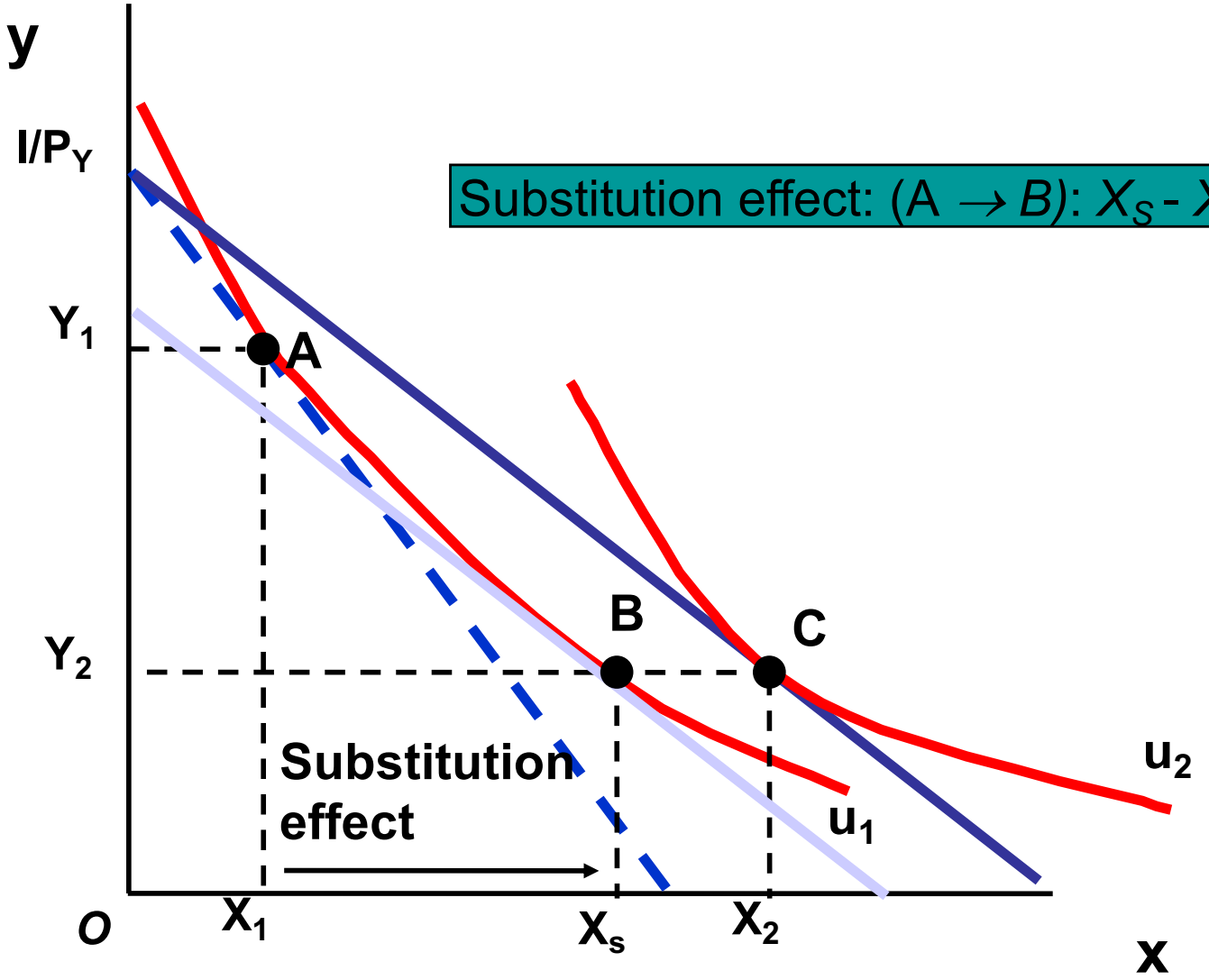
(2) Consumer's purchasing power increases due to the fact that he can buy the same amount of the good for less money and expend the saved income on the same good or on others. The effect caused by the variation of the purchasing power is called INCOME EFFECT.

Substitution and income effects

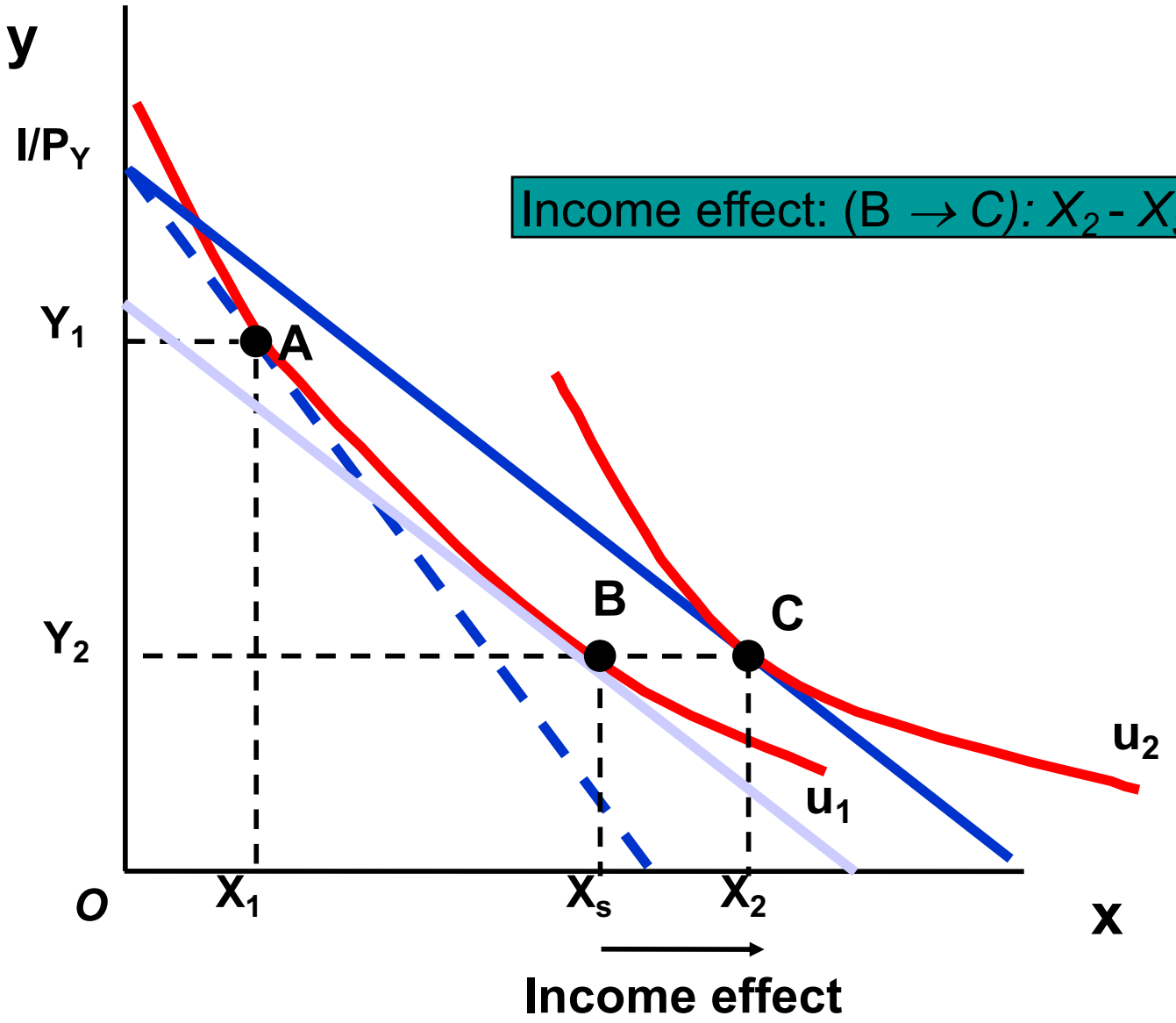
The substitution effect is the variation experienced by the demand of a good when its price changes and the utility level keeps constant (Hicks viewpoint).

The income effect is the variation experienced by the demand of a good when the purchasing power changes and the relative price keeps constant.

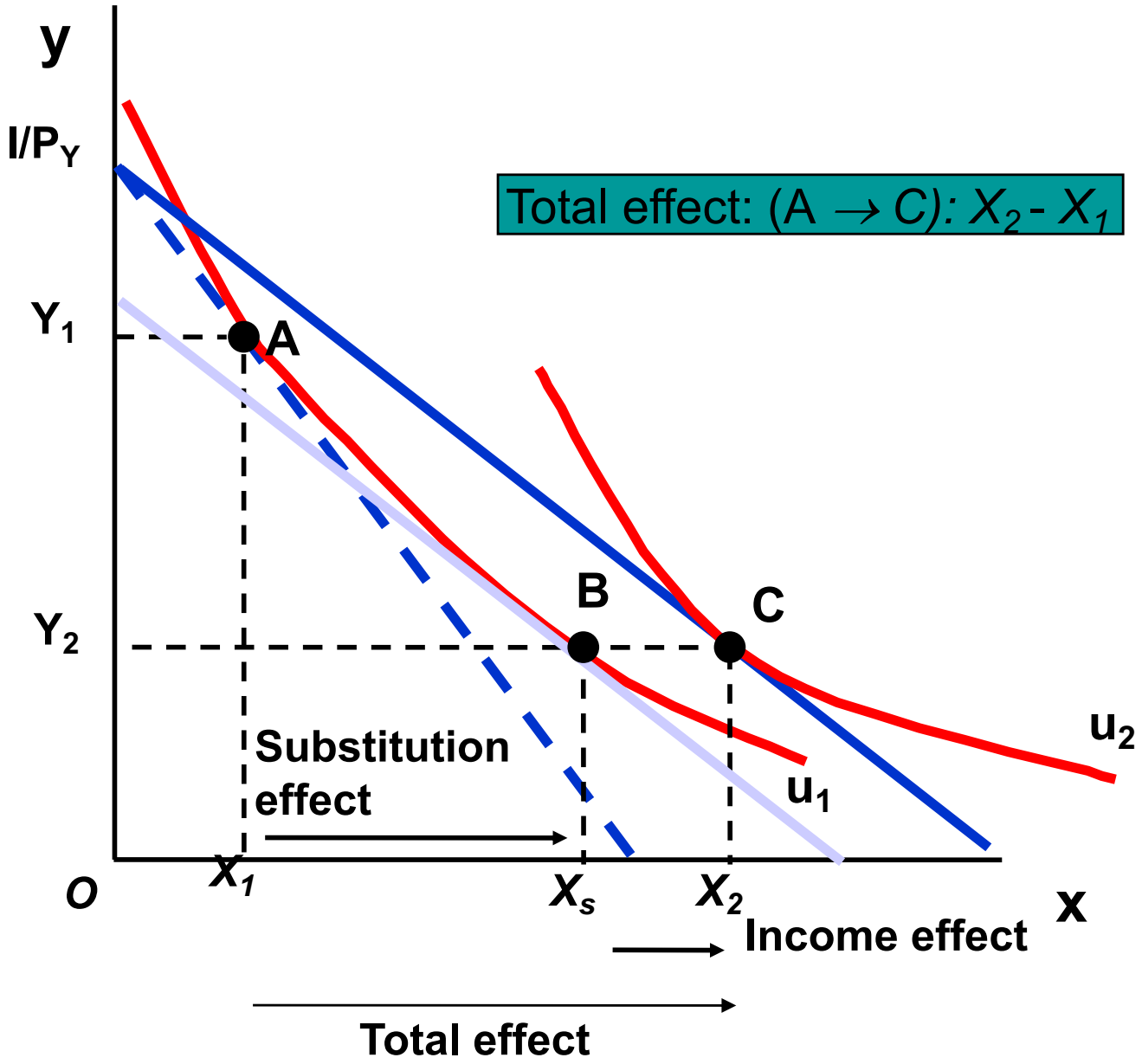
Substitution effect after a decrease in p_x



Income effect after a decrease in p_x



Total effect after a decrease in p_x



Example: Cobb-Douglas utility function

Data: $u(x,y) = x^{½} y^{½}$; $p_x = 8$, $p_y = 2$, $I = 16$

Calculate the SE and the IE of a decrease of price of x: $p'_x = 2$

Solution:

Initial situation (1)

$x_1 = I / (2\ p_x) = 1$

$y_1 = I / (2\ p_y) = 4$

$u_1 = 2$

Final situation (2)

$x_2 = I / (2\ p'_x) = 4$

$y_2 = I / (2\ p_y) = 4$

$u_2 = 4$

Example: Cobb-Douglas utility function

Now, we must calculate the compensated change (point B in the graph). The "intermediate" bundle S must satisfy:

(a) Tangency condition: $MRS = p'_x / p_y \rightarrow y_S / x_S = 1$

(b) Compensated utility: $u_S = u_1 \rightarrow x_S^{1/2} y_S^{1/2} = 2$

Solving, $x_S = 2 \quad y_S = 2$

Substitution effect	+	Income effect	=	Total effect
$x_S - x_1 = 2 - 1 = 1$		$x_2 - x_S = 4 - 2 = 2$		$x_2 - x_1 = 4 - 1 = 3$

Signs of SE and IE

The sign of the substitution and income effects identify the directions of variation of the price and demand of a good (we are NOT saying that mathematically they are greater or less than zero).

Signs of SE and IE

The SE is always negative: The price and of the demand of the good move in opposite directions. If the price increases, the demand decreases (that is, SE < 0), and vice versa, if the price decreases, the demand increases (that is, SE > 0).

The sign of the IE depends on whether the good is normal or inferior:

- If the good is normal, the decrease in the purchasing power of income caused by a price increase leads to a decrease in the demand. (The sign of the IE is negative).
- If it is an inferior good, the decrease in the purchasing power of income caused by a price increase leads to an increase in the demand. (The sign of the IE is positive).

SE and IE in a normal good

When a good is normal, SE and IE act in the same way reinforcing each other.

Suppose a decrease in the price of good x:

SE will be greater than 0 (a decrease in price implies an increase in the demand due to the fact that the good is relatively cheaper).

IE will be greater than 0 (a decrease in price increases the purchasing power of income, which increases demand because x is a normal good).

Therefore,

$$TE = SE + IE > 0.$$

SE and IE in an inferior good

When a good x is inferior, SE and IE act in opposite directions. Consequently, the sign of the total effect depens on the sizes of the substitution and income effects, and cannot be determined *a priori*.

Suppose a decrease in the price of good x:

SE is greater than 0 (a decrease in prices implies an increase in the demand due to the fact that the good is relatively cheaper).

IE is less than 0 (a decrease in price increases the purchasing power of income, which decreases demand because x is an inferior good).

Therefore,

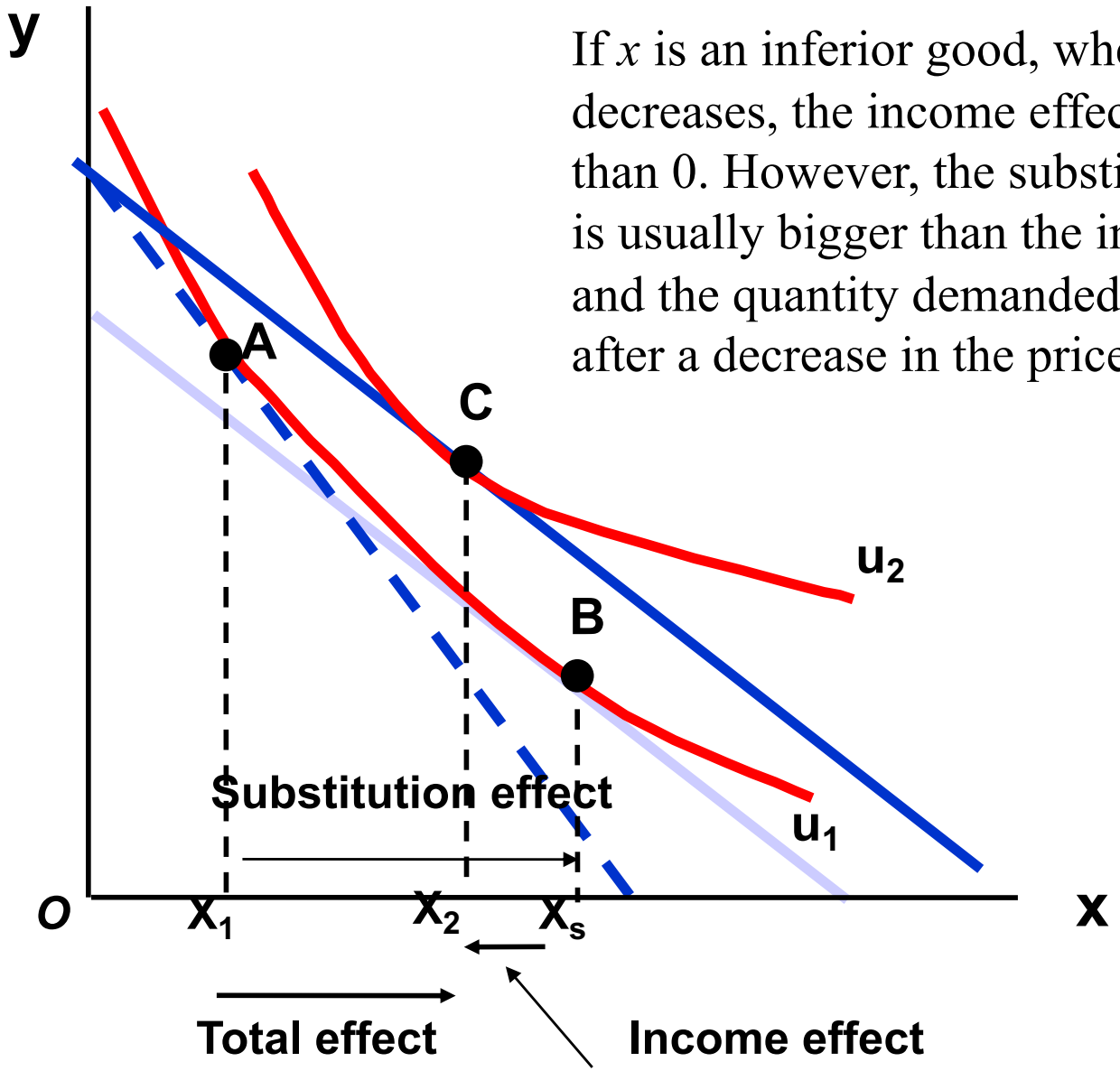
$$TE(??) = SE(>0) + IE(<0).$$

SE and IE in an inferior good

If x is good an inferior, the sign of the total effect depends on which of the two effects is bigger (substitution or income ones).

The income effect is seldom bigger than the substitution effect, and therefore in general the total effect of a decrease in the price of a good is bigger than 0. In other words, the demand is decreasing with respect to the price.

SE an IE in an inferior good



If x is an inferior good, when its price decreases, the income effect is lower than 0. However, the substitution effect is usually bigger than the income one and the quantity demanded increases after a decrease in the price of a good.

Exception: Giffen goods (increasing demand with the price)

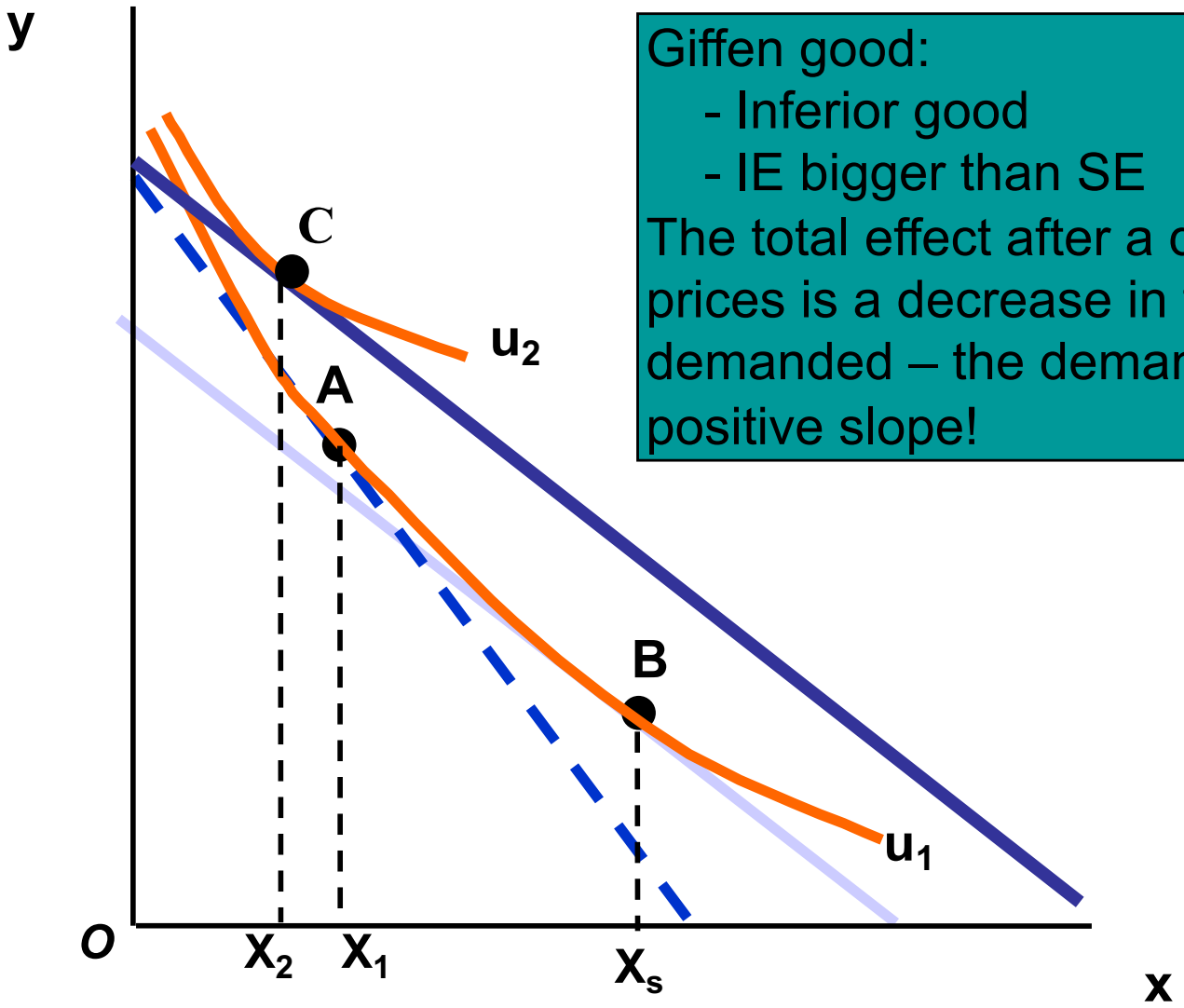
In theory, the income effect may be large enough to make the demand function of an inferior good to have a positive slope.

A Giffen good is an inferior good whose demand increases with the price of the good (the income effect is larger than the substitution effect).

Exception: Giffen goods (increasing demand with the price)

During the starvation of XIX century in Ireland, Robert Giffen noticed that the price of potatoes increased and so did the quantity demanded.

SE and IE in a Giffen good



Giffen good:
- Inferior good
- IE bigger than SE

The total effect after a decrease in prices is a decrease in the quantity demanded – the demand curve has positive slope!