Consumer Theory

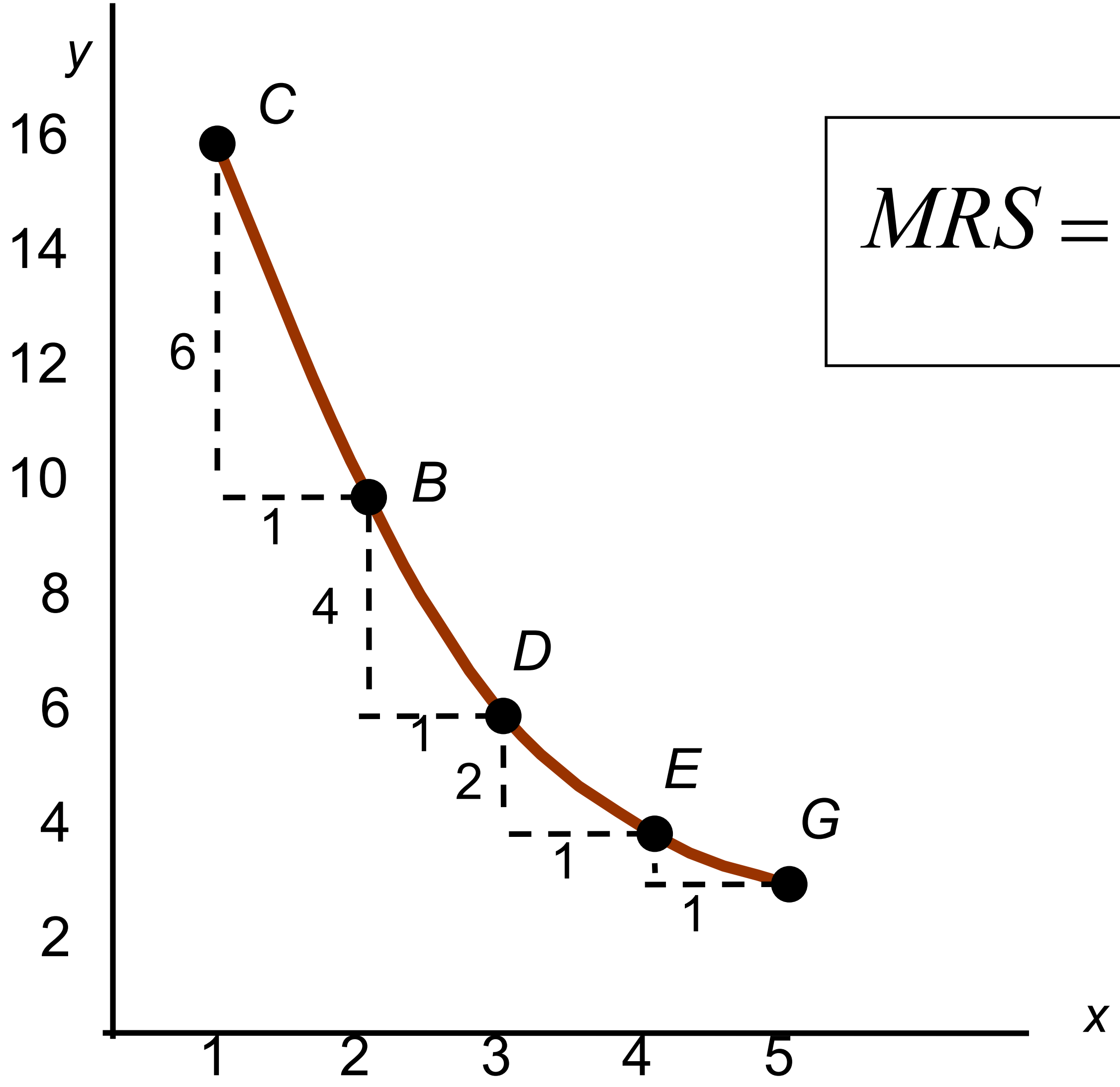
The Marginal Rate of Substitucion

The Marginal Rate of Substitution

The marginal rate of sustitution (*MRS*) is the value of a unit of good *x* measured in units of good *y*; that is, it is the maximum quantity of good *y* a consumer is willing to give up in order to get an additional unit of good *x*, or alternatively, it is the number of units of good *y* needed to compensate the consumer from losing one unit of good *x*.

The MRS is a function $MRS: \Re^2_+ \rightarrow \Re$. The value $MRS(x,y)$ is not generally constant, but depends on the relative scarcity of each good in the bundle (x,y).

The Marginal Rate of Substitution



$$MRS = \frac{-\Delta y}{\Delta x}$$

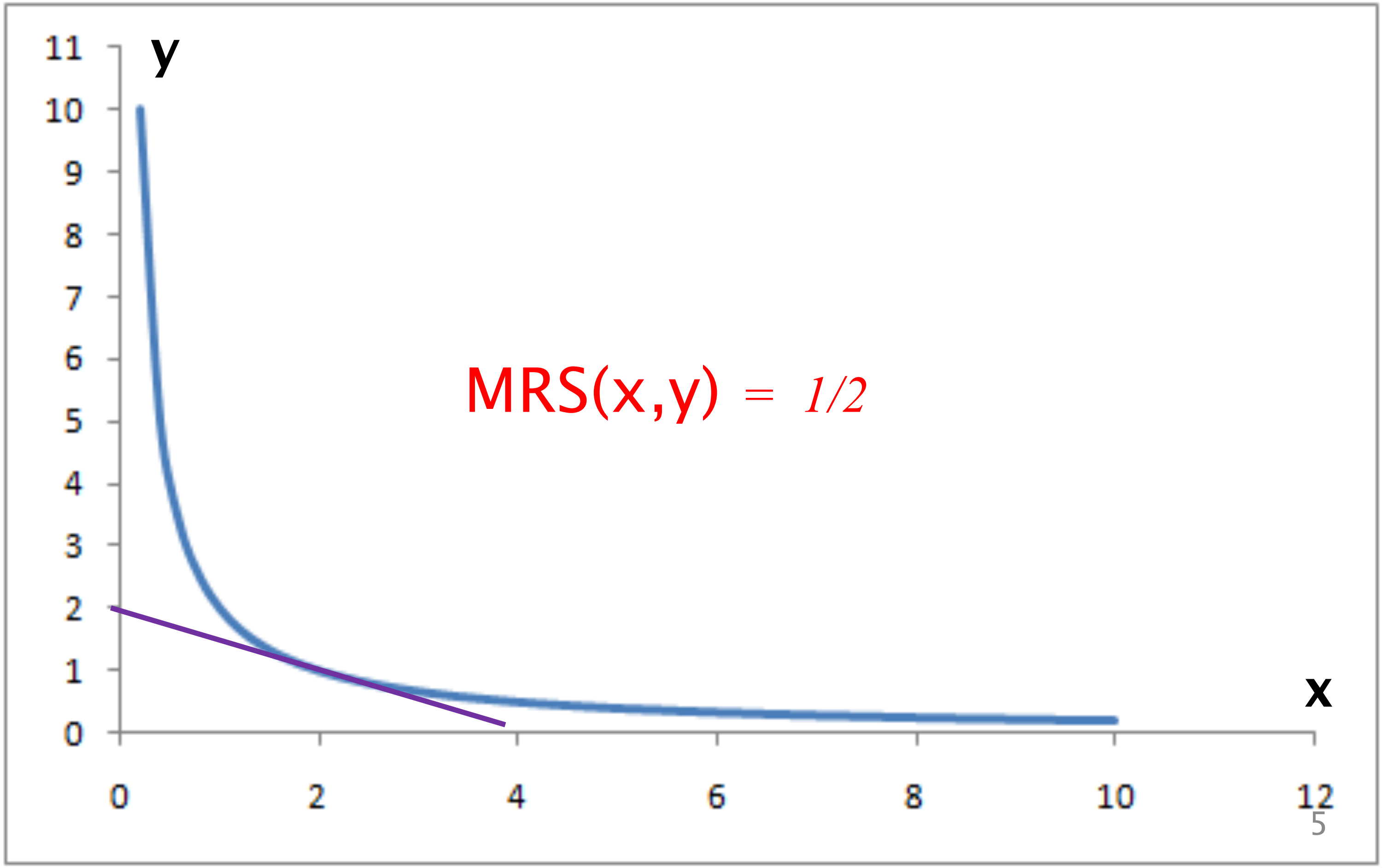
The Marginal Rate of Substitution

In order to make the "MRS" a more useful concept, and facilitate its calculation, we define the *MRS(x,y)* as the quantity of good *y* needed to compensate the consumer from a loss of one *infinitesimal* unit of good *x*, so that the consumer maintains the level of welfare he has with the bundle *(x,y)*.

That is, the *MRS(x,y)* is the value to the consumer of an infinitesimal unit of good *x*, given in units of good *y*, when the consumer's bundle is *(x,y)*.

The Marginal Rate of Substitution

The MRS(x,y) is the absolute value of the slope of the line tangent to the indifference curve at (x,y).



The MRS: Examples

1. $u(x,y) = xy$

Denote by $u(x,y) = xy = u^*$ the utility level at the consumption bundle (x,y). Then

$$u^* = xy \rightarrow y = f(x) = u^*/x.$$

Therefore

$$f'(x) = -u^*/x^2.$$

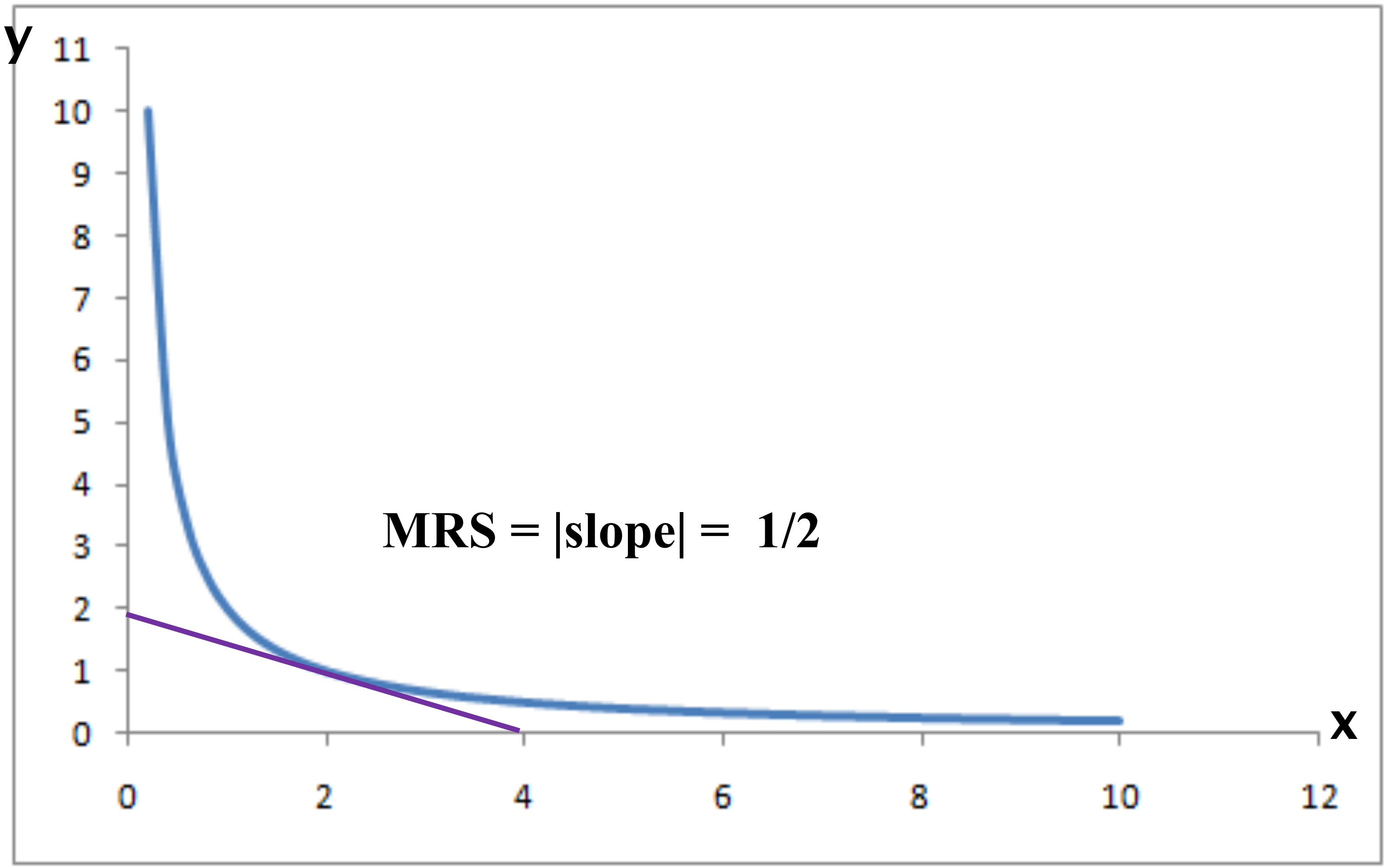
Substituting $u^*=xy$ we obtain

$$MRS(x,y) = |-xy/x^2| = y/x.$$

Evaluating the MRS at (2,1) yields

$$MRS(2,1) = 1/2.$$

The MRS: Examples



The MRS: Examples

2. $u(x,y) = 2x + y$

Denote by $u^* = 2x + y = u^*$ the utility level at the consumption bunde (x,y). Then

$$u^* = 2x + y \rightarrow y = f(x) = u^* - 2x.$$

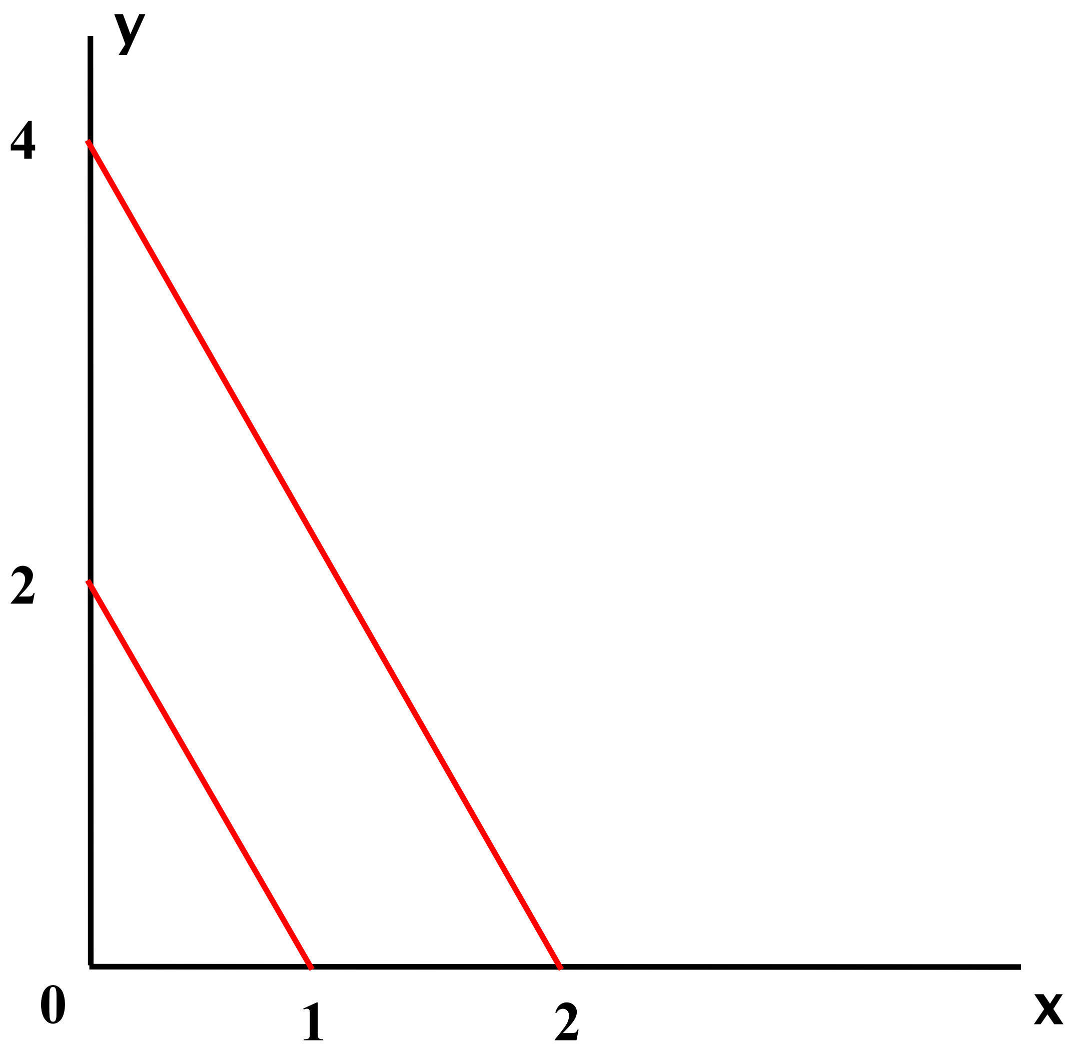
Therefore

$$MRS(x,y) = |f'(x)| = 2.$$

In this case, the MRS is a constant and equal to 2.

The MRS: Examples

3. The goods x and y are perfect substitutes.



The MRS: Examples

3. $u(x,y) = \min\{x,2y\}$

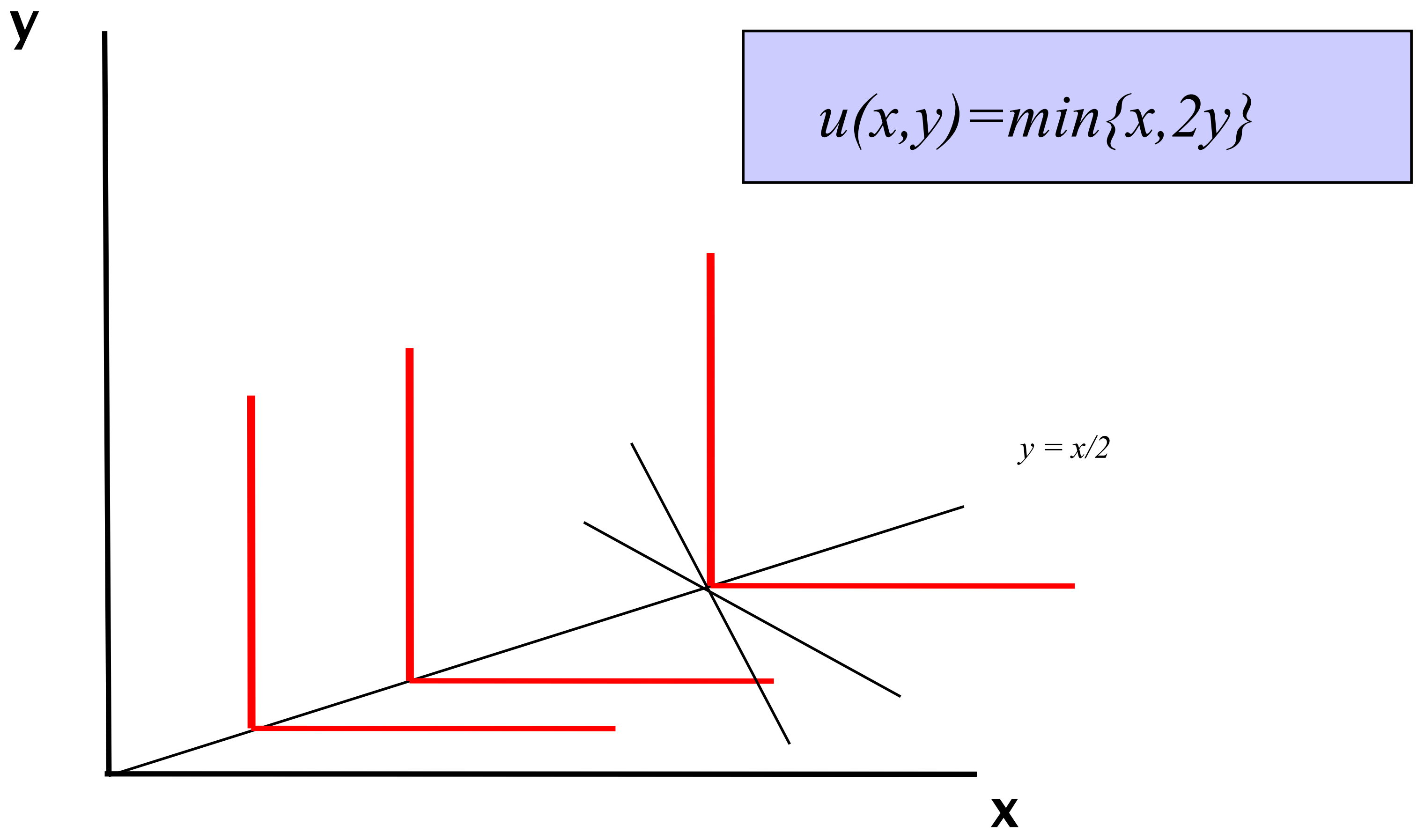
This utility function is not differentiable at (x,y) when $x \leq 2y$. For these points, the MRS is not defined.

At points (x,y) such that $x > 2y$, we have

$$MRS(x,y)=0.$$

The MRS: Examples

3. MRS(x,y) = 0 if $y < x/2$, and MRS(x,y) is not defined if $y \geq x/2$.



Calculating the MRS

We can find an expression for the $MRS(x,y)$ without knowing the function $y = f(x)$ that defines the indifference curve.

In order to calculate the $MRS(x_0,y_0)$, we start from the equation that defines the indifference curve at point (x_0,y_0)

$$u(x,y) = u_0, \qquad (*)$$

where $u(x_0,y_0) = u_0$.

The Implicit Function Theorem establishes conditions that guarantee that this equation defines a function around the point (x_0,y_0), and under these conditions ensures that the derivative of this function can be obtained by total differentiation.

Calculating the MRS

Denoting the partial derivatives of $u(x,y)$ with respect to x and y as u_x and u_y, respectively, and taking the total derivative of the equation (*), we obtain

$$dx\, u_x + dy\, u_y = 0.$$

Hence the derivative of the function defined by equation (*) is

$$|dy/dx| = |-u_x/u_y| = u_x/u_y$$

Therefore $MRS(x_0,y_0)$ is obtained by evaluating this expression at (x_0,y_0):

$$MRS(x_0,y_0) = u_x(x_0,y_0)/u_y(x_0,y_0)$$

Calculating the MRS

We apply this formula to examples 1 and 2 above.

1. $u(x,y)=xy.$

We have $u_x= \partial U/\partial x=y$, and $u_y= \partial U/\partial y=x$. Hence

$$MRS(x,y)= u_x/u_y = y/x.$$

2. $u(x,y)=2x+y$

We have $u_x= \partial U/\partial x =2$, and $u_y= \partial U/\partial y =1$. Hence

$$MRS(x,y)= u_x/u_y = 2/1= 2.$$

The Marginal Rate of Substitution

The MRS defines completely the consumer's preferences:

- The MRS allows to reproduce the consumer's indifference map.
- La MRS is invariant to monotonic transformations of the utility function: If $f: R \rightarrow R$ satisfies $f' > 0$ and v and u are such that

$$v(x,y)=f(u(x,y)),$$

then

$$RMS_v(x,y) = v_x / v_y = f'u_x / f'u_y = u_x / u_y = RMS_u(x,y).$$