Theory of the Firm

The Competitive Firm

The Competitive Firm

In a competitive maket, the production level of a firm has a neglegible impact of the market price. Hence, the firm may take the market price as given; i.e., the firm acts as a *price taker*.

The revenue to the firm if it produces *q units is*

$$R(q) = pq$$

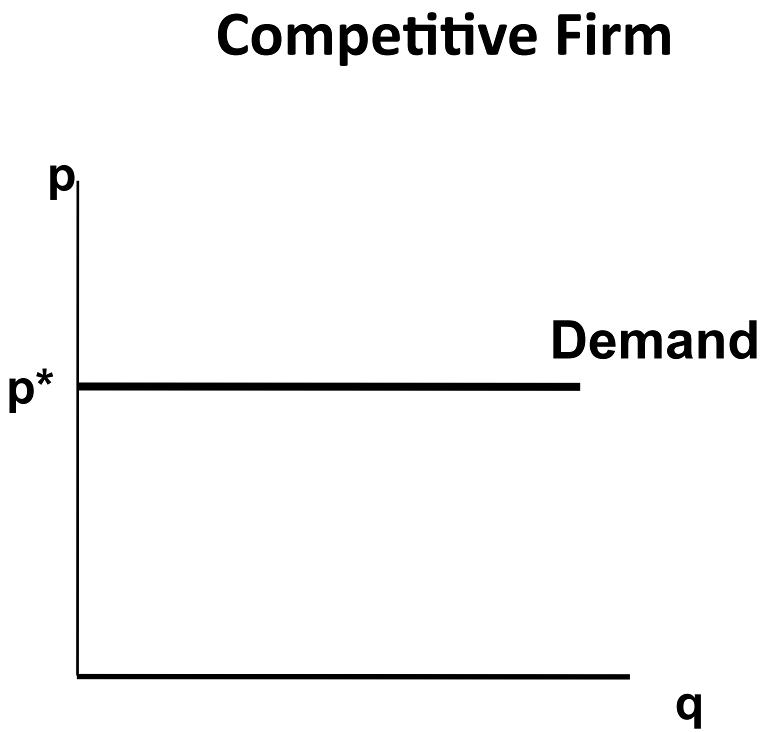
and the firm's profit is

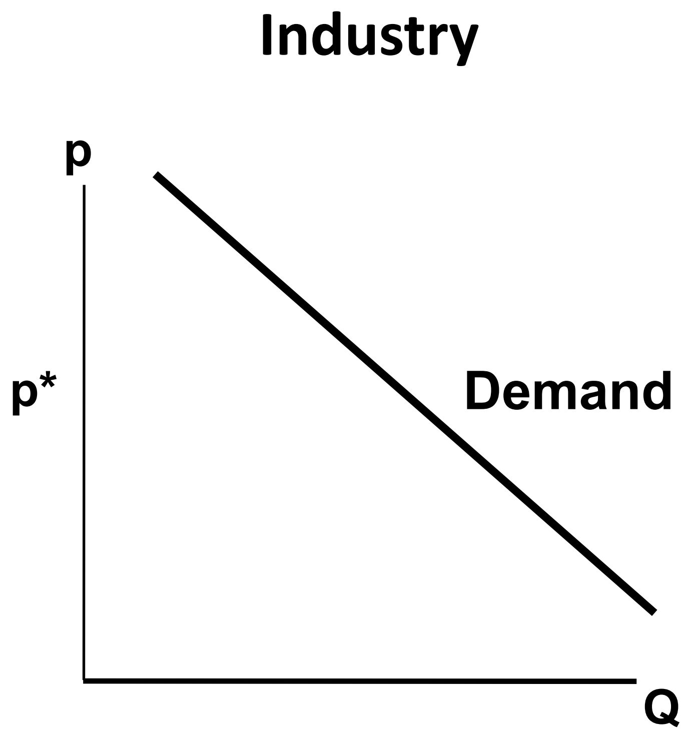
$$\pi(q) = pq - C(q)$$

Hence the firm's problem is

$$\mathrm{Max}_{q \geq 0}\ \pi(q).$$

The Competitive Firm





Demand curve of the firm= Average/Marginal Revenue curve

The Competitive Firm

In order for $q^* > 0$ to solve the firm's problem it must satisfy

1. First Order Condition (FOC): $\pi'(q) = 0$; that is,

$$p = MC(q).$$

2. Second Order Condition (SOC): $\pi''(q) \leq 0$, that is,

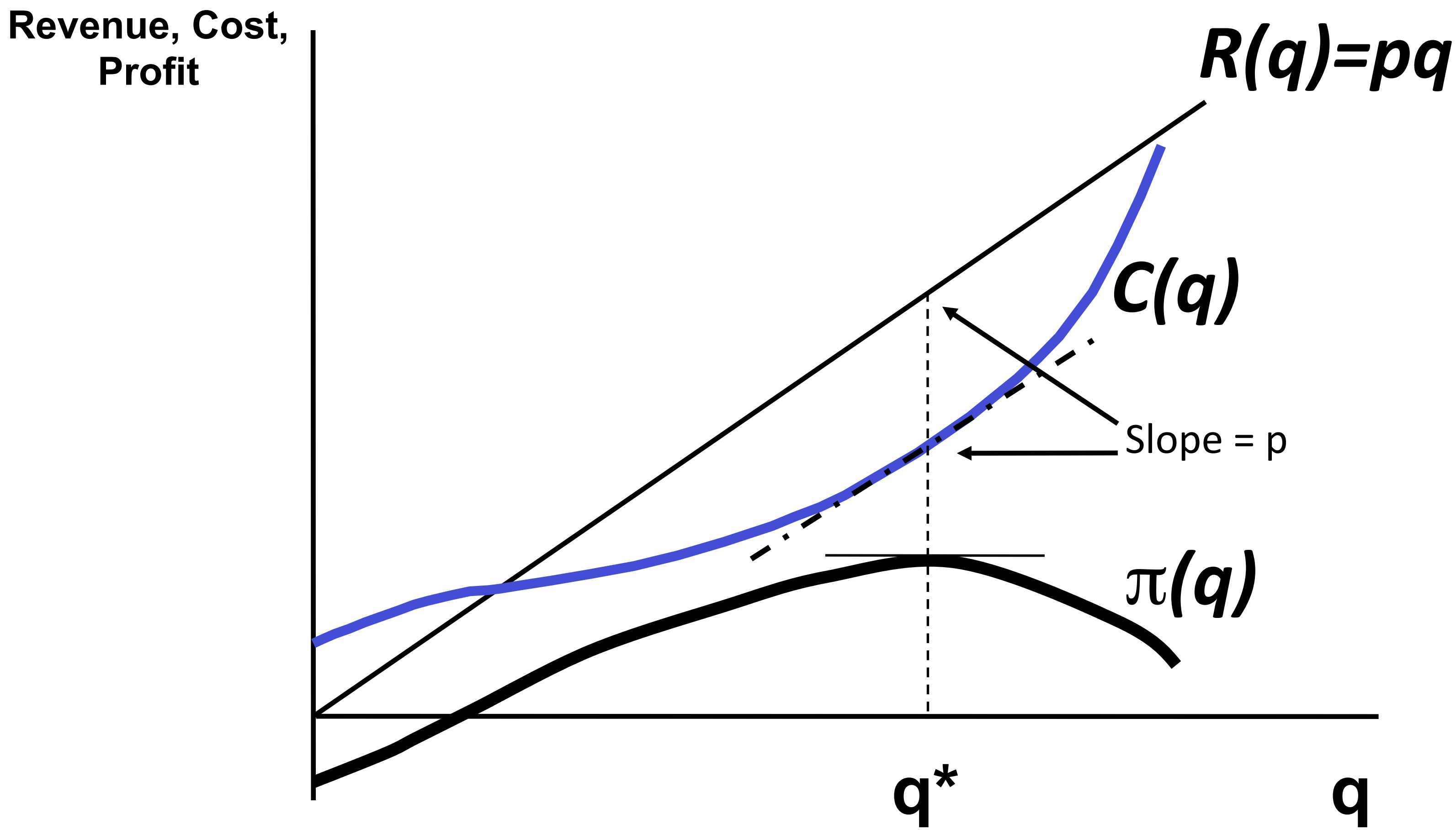
$$MC'(q) \geq 0.$$

3. Moreover, for the solution to be *interior* (i.e., $q^* > 0$), the it must satisfy

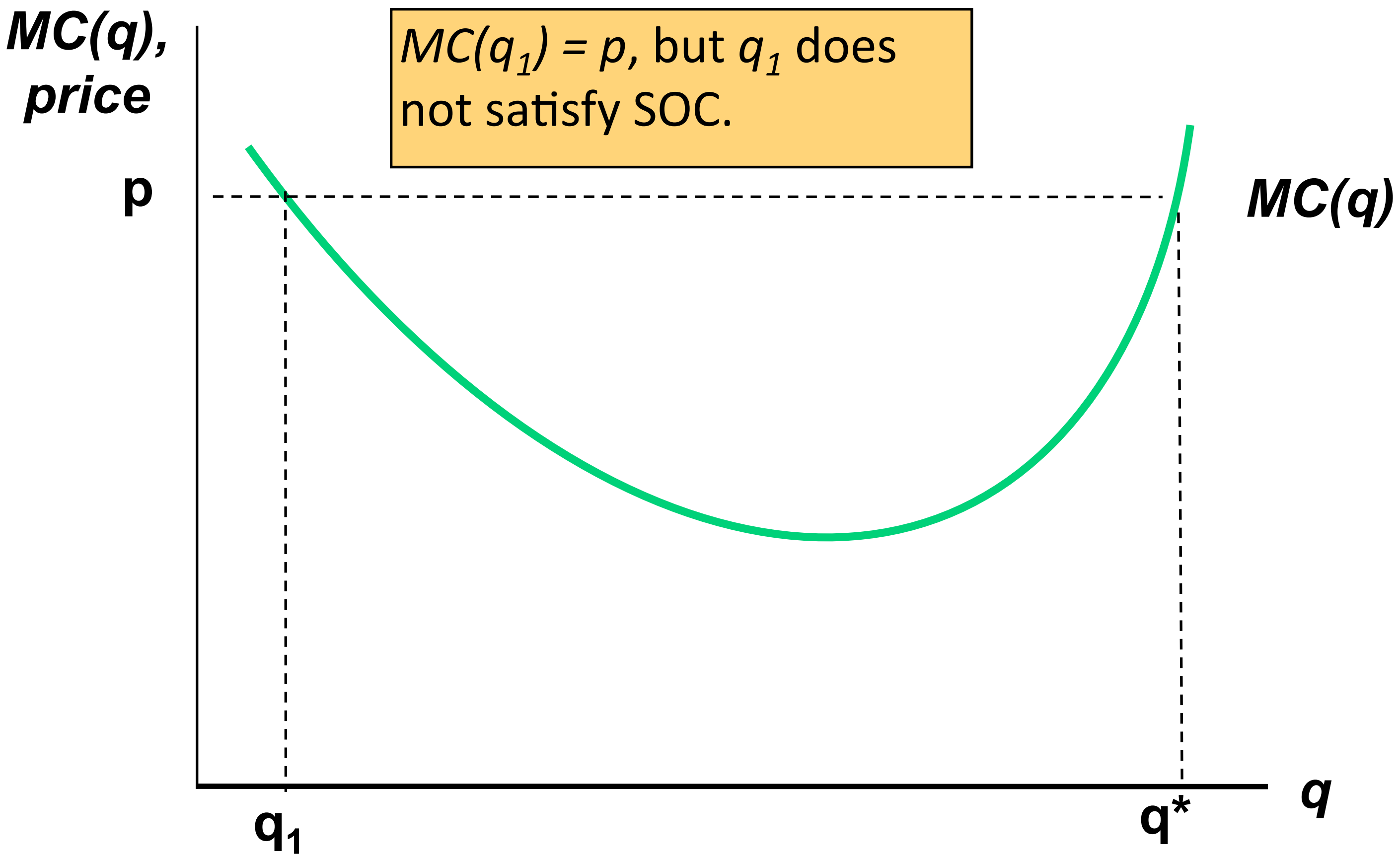
$$\pi(q^*) \geq \pi(0). \quad \text{(Shutdown Condition)}$$

(If $\pi(q^*) < \pi(0)$, then the optimal output is $q = 0$.)

The Competitive Firm: FOC



The Competitive Firm: SOC



The Competitive Firm

Shutdown Condition: $\pi\ (q^*) \geq \pi\ (0)$

Long run (there are no FC):

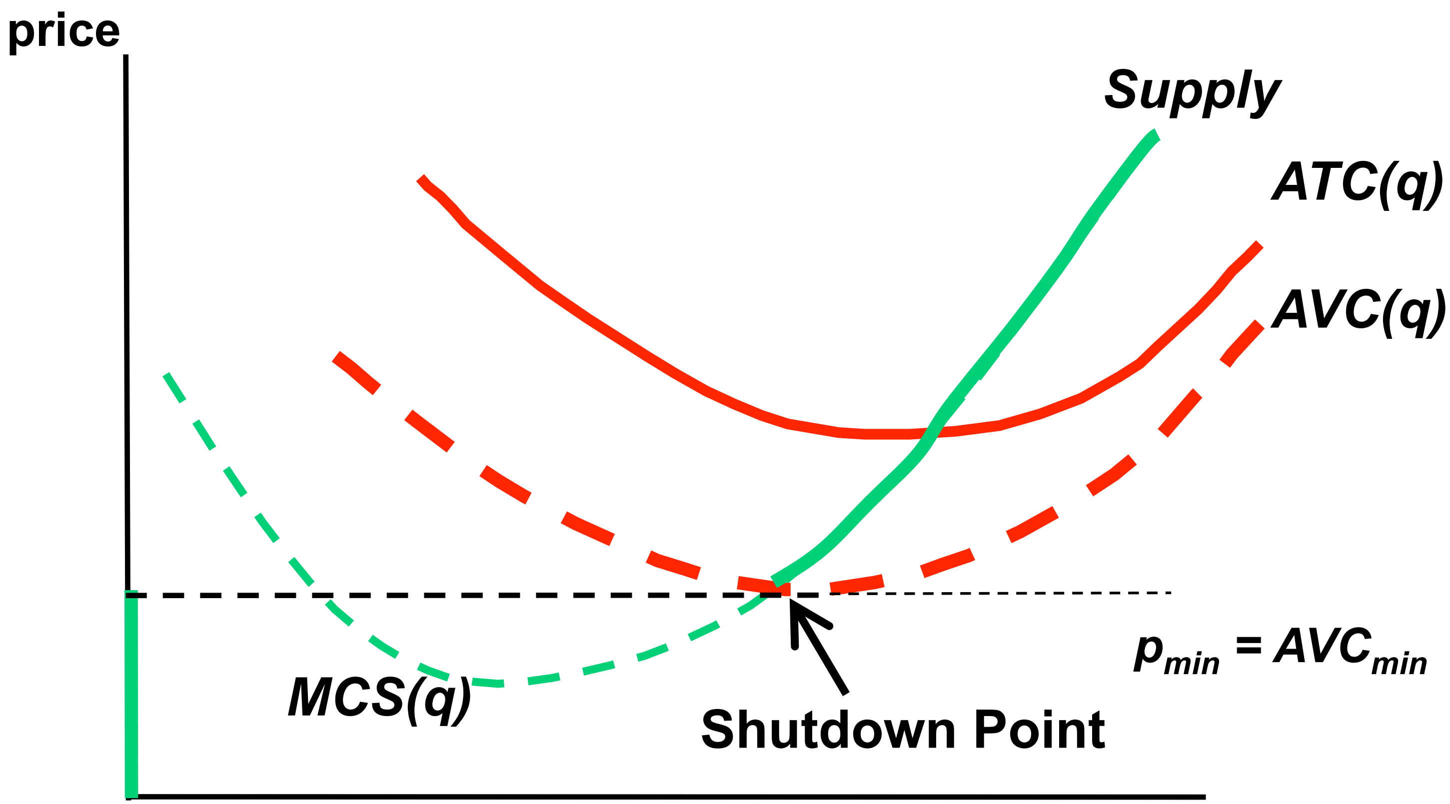
$$pq^* - C(q^*) \geq 0 \Rightarrow p \geq C(q^*)/q^* = AC(q^*).$$

Short run (there are FC):

$$pq^* - VC(q^*) - FC \geq - FC \Leftrightarrow p \geq VC(q^*)/q^* = AVC(q^*)$$

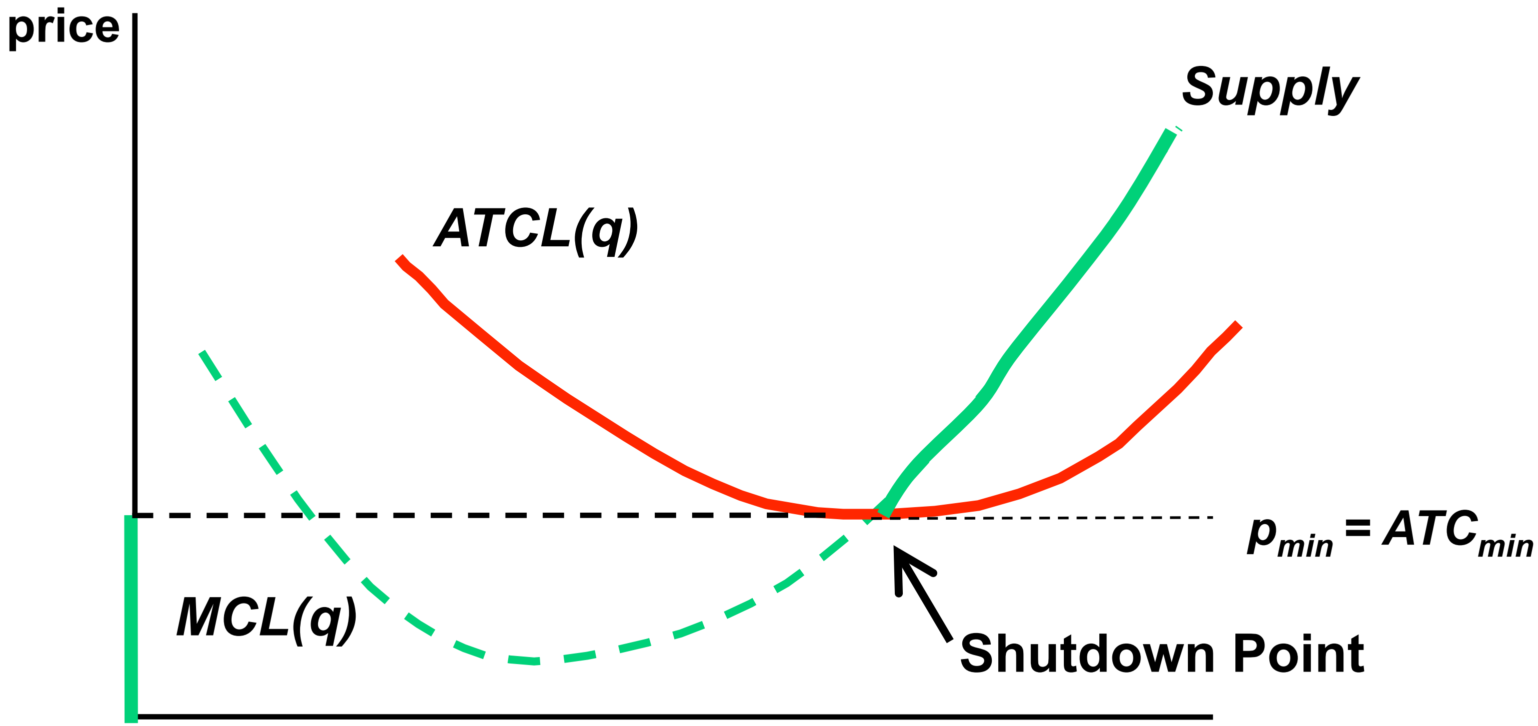
The Competitive Firm: SC

Short Run (there are FC)

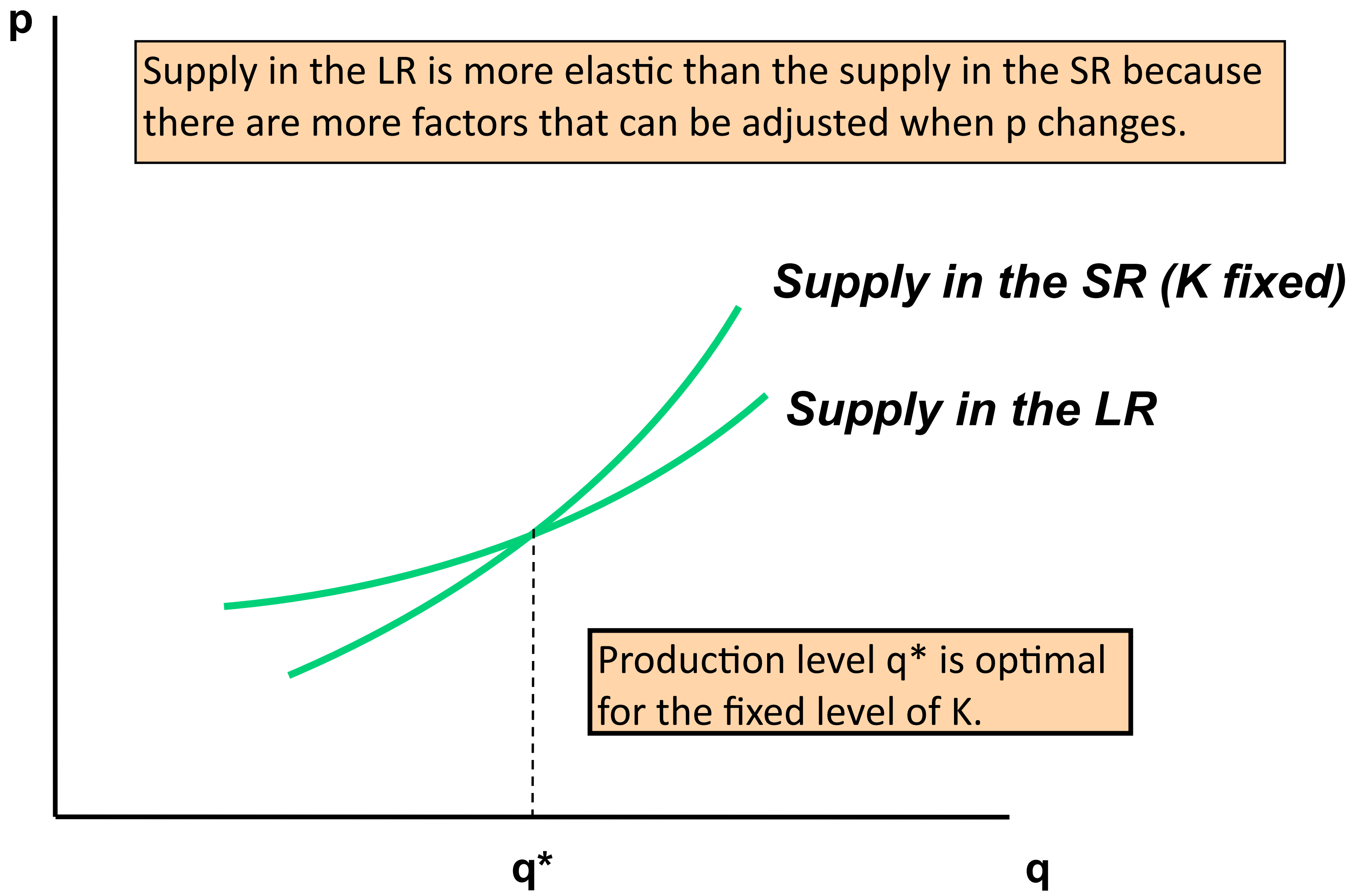


The Competitive Firm

Long Run (no FC)



The Competitive Firm



Supply of the Competitive Firm: Example

Find the supply of a competitive firm with cost function $C(q) = 100 + q^2$.

- First Order Condition for profit maximization:

$$p = MC(q) \Leftrightarrow p = 2q \Leftrightarrow q = p/2$$

- Second Order Condition:

$$dMC(q)/dq = 2 > 0$$

- Shutdown condition:

$$p(p/2) - (p/2)^2 - 100 \geq -100 \Leftrightarrow p^2/4 \geq 0,$$ it holds for every $p \geq 0$

Therefore, the supply of the firm is

$$q^S = S(p) = p/2$$

Supply of the Competitive Firm: Example

Find the supply of a competitive firm with cost function

$C(q) = (q^3/3) - 2q^2 + 4q + 10.$

- First Order Condition for profit maximization:

$$p = MC(q) \Leftrightarrow p = q^2 - 4q + 4$$

- Second Order Condition:

$$dMC(q)/dq \geq 0 \Leftrightarrow 2q - 4 \geq 0 \Leftrightarrow q \geq 2$$

- Shutdown condition:

$$(q^2 - 4q + 4)q - (q^3/3) + 2q^2 - 4q - 10 \geq -10 \Leftrightarrow q \geq 3$$

Notice that shutdown condition is stricter than SOC.

Supply of the Competitive Firm: Example

Thus, the supply function of the firm is

$$p = q^2 - 4q + 4 \Leftrightarrow q^s = S(p) = 2 + \sqrt{p}$$

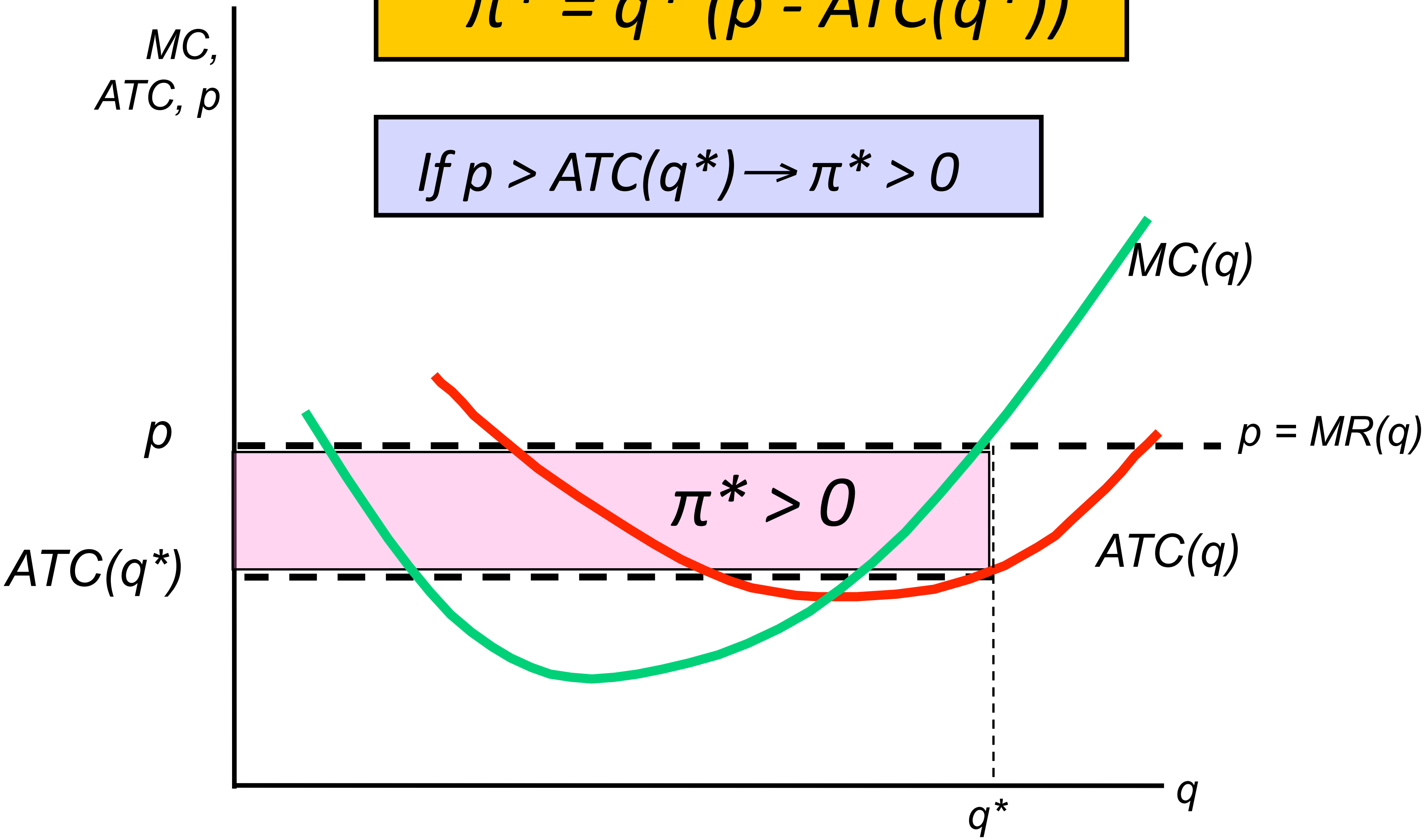
As *S(p)* has to be greater or equal to 3,

$$S(p) = \begin{cases} 2 + \sqrt{p} & if \;\; p \geq 1 \\ 0 & if \;\; p < 1 \end{cases}$$

Profits of the Competitive Firm

$\pi^* = q^* (p - ATC(q^*))$

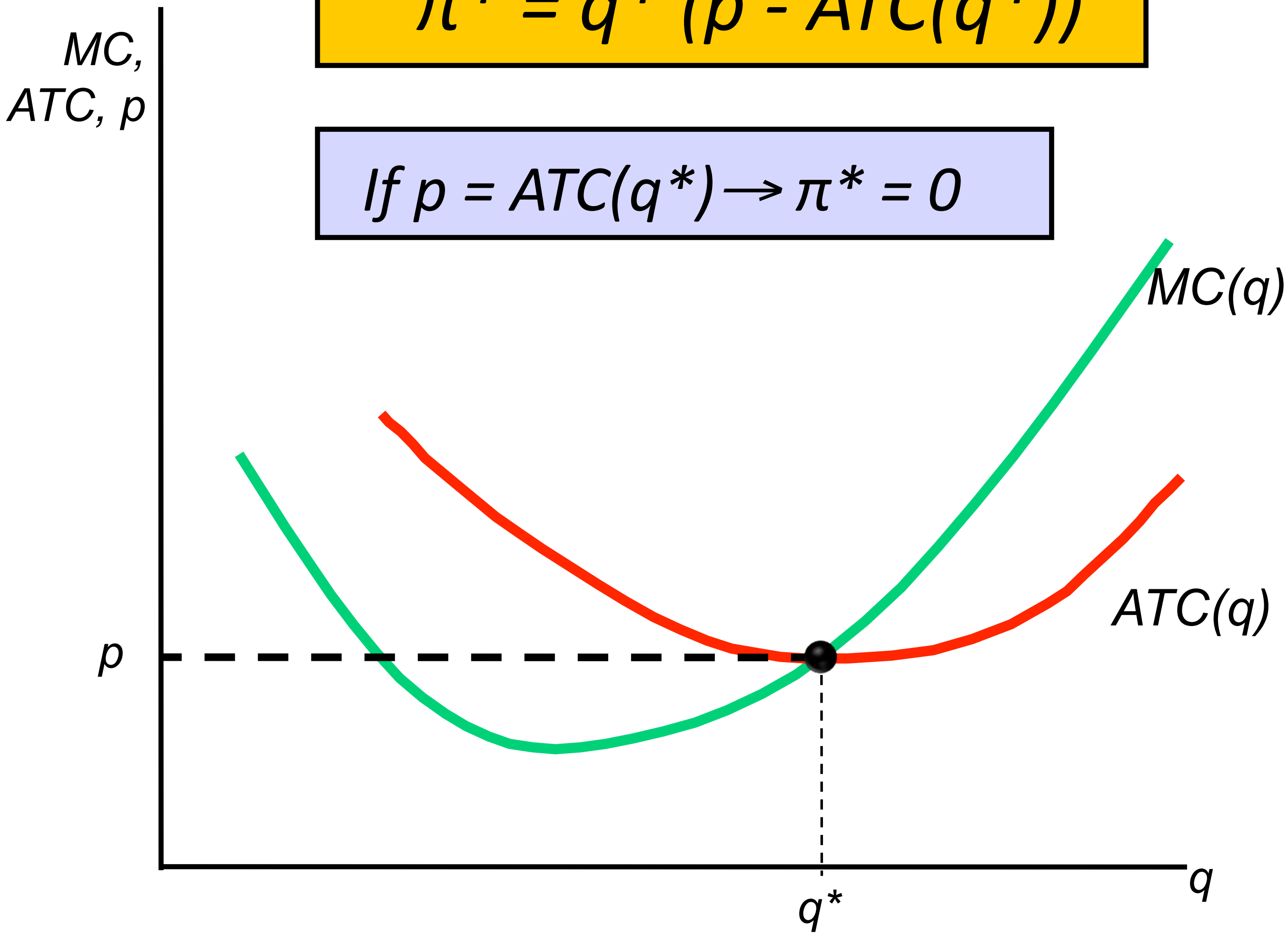
If $p > ATC(q^*) \rightarrow \pi^* > 0$



The Competitive Firm

$$\pi^* = q^* (p - ATC(q^*))$$

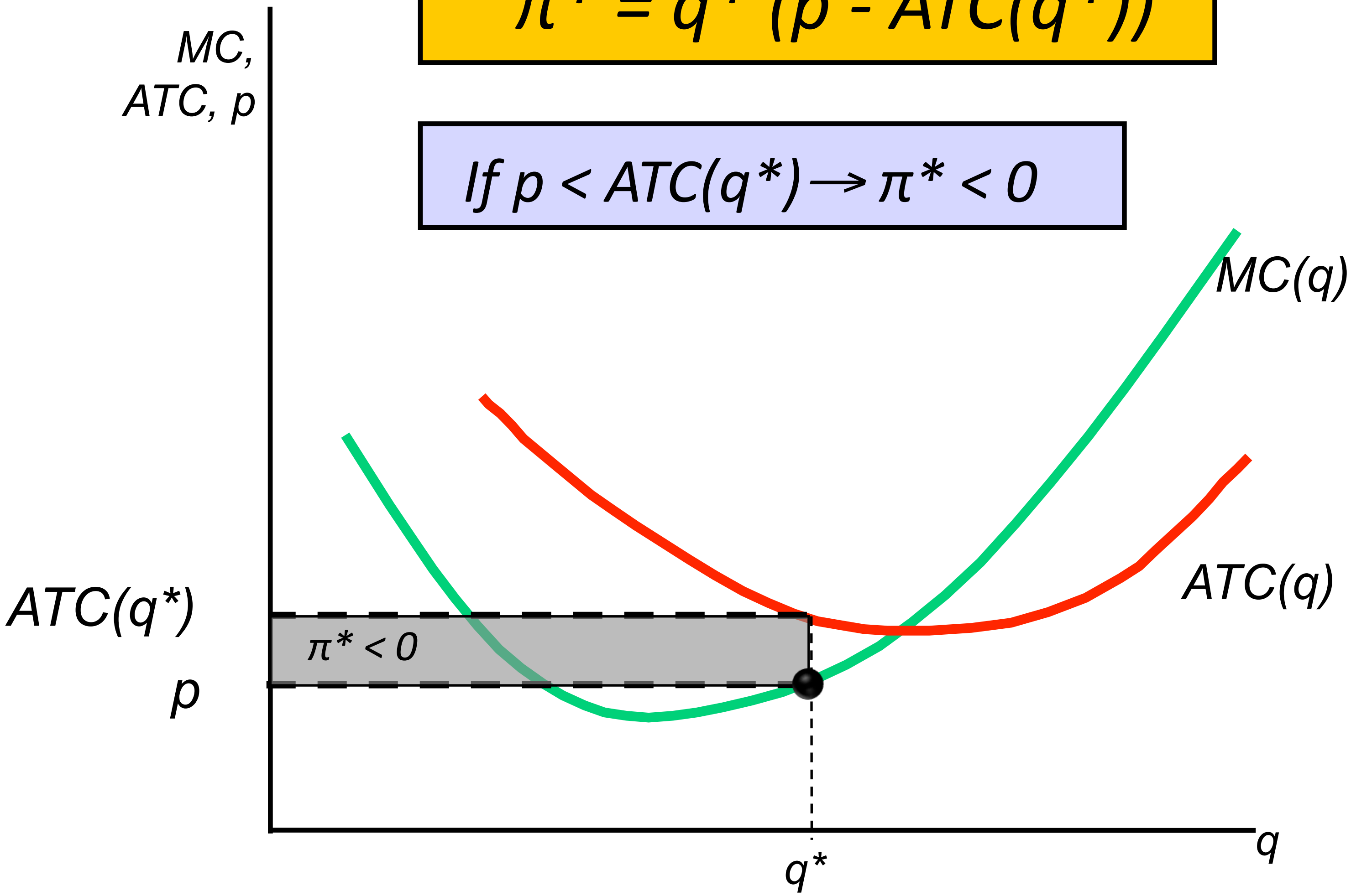
If $p = ATC(q^*) \rightarrow \pi^* = 0$



The Competitive Firm

$\pi^* = q^* (p - ATC(q^*))$

If $p < ATC(q^*) \rightarrow \pi^* < 0$



Input Demands: Long Run

To find out the input demand functions, we can plug the supply function of the competitive firm into the conditional demands, and in this way we get input demands which only depend on prices (input prices, and price of the good):

$$L^*(Q,w,r) = L^*(S(p),w,r) = L^*(p,w,r)$$

$$K^*(Q,w,r) = K^*(S(p),w,r) = K^*(p,w,r)$$

Demand of Inputs - Long Run

Alternatively, we may reconsider the profit maximization problem without using the cost function *C(q)*:

$$\max_{\{L,K\}} pF(L,K) - wL - rK$$

And we can obtain the input demand functions from the first order conditions:

$$w = p \cdot MP_L(L,K)$$
$$r = p \cdot MP_K(L,K)$$

Input Demands - Long Run: Example

$F(L,K) = AL^{1/3}K^{1/3}$

FOC from the profit maximization problem:

$(1) p(A/3)L^{-2/3}K^{1/3} = w$

$(2) p(A/3)L^{1/3}K^{-2/3} = r$

Input demands:

$L^* = (Ap)^3/(27w^2r)$

$K^* = (Ap)^3/(27wr^2)$

Input Demands - Short Run

In the short run, the level of K is fixed to K_0. Input demand function in the short run is the solution to:

$$\max_{\{L\}} pF(L, K_0) - wL - rK_0$$

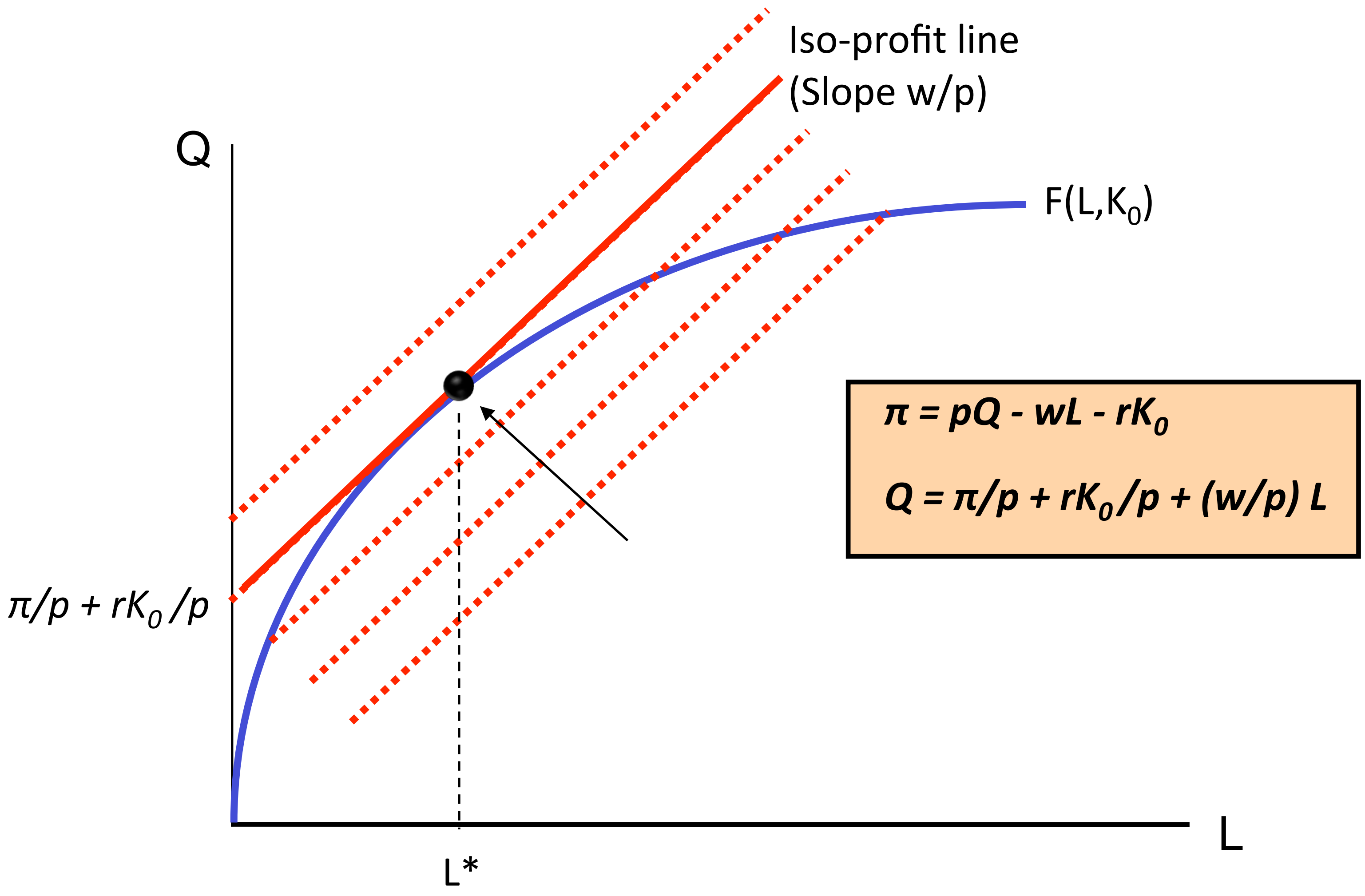
First Order Condition:

$$w = p \cdot MP_L(K_0, L)$$

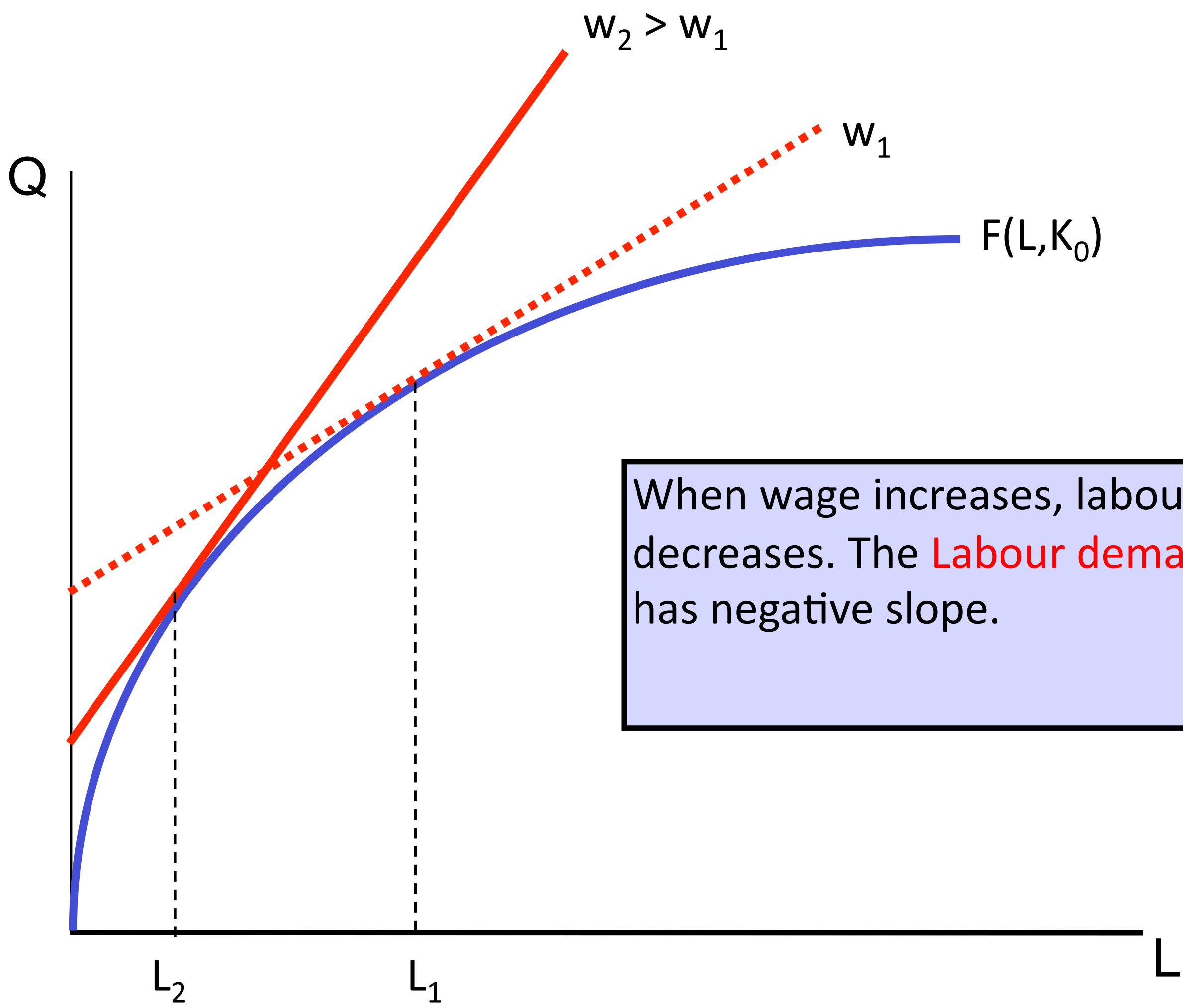
Solution:

$$L^* = L(p, w, r)$$

Input Demands - Short Run

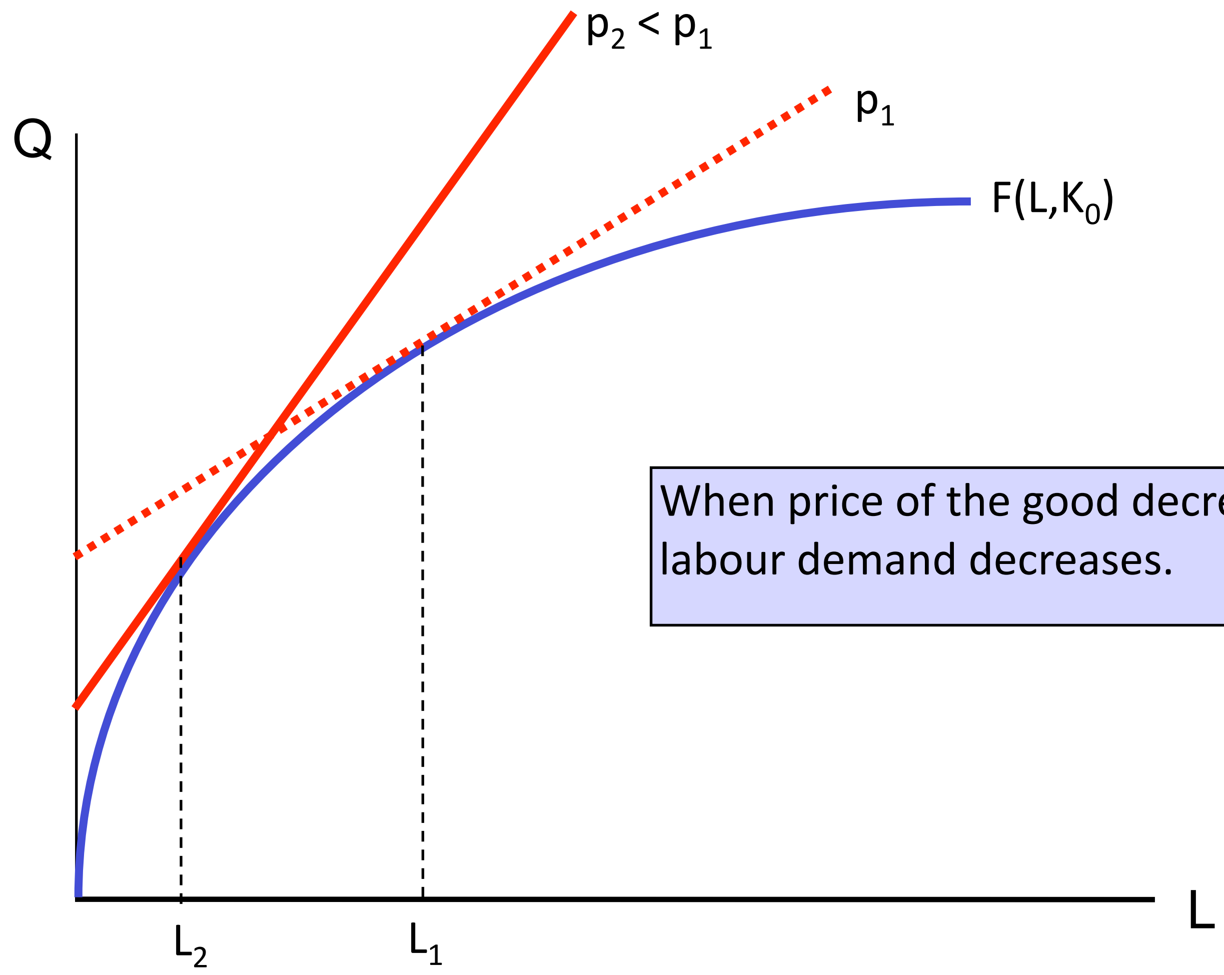


Labour Demand



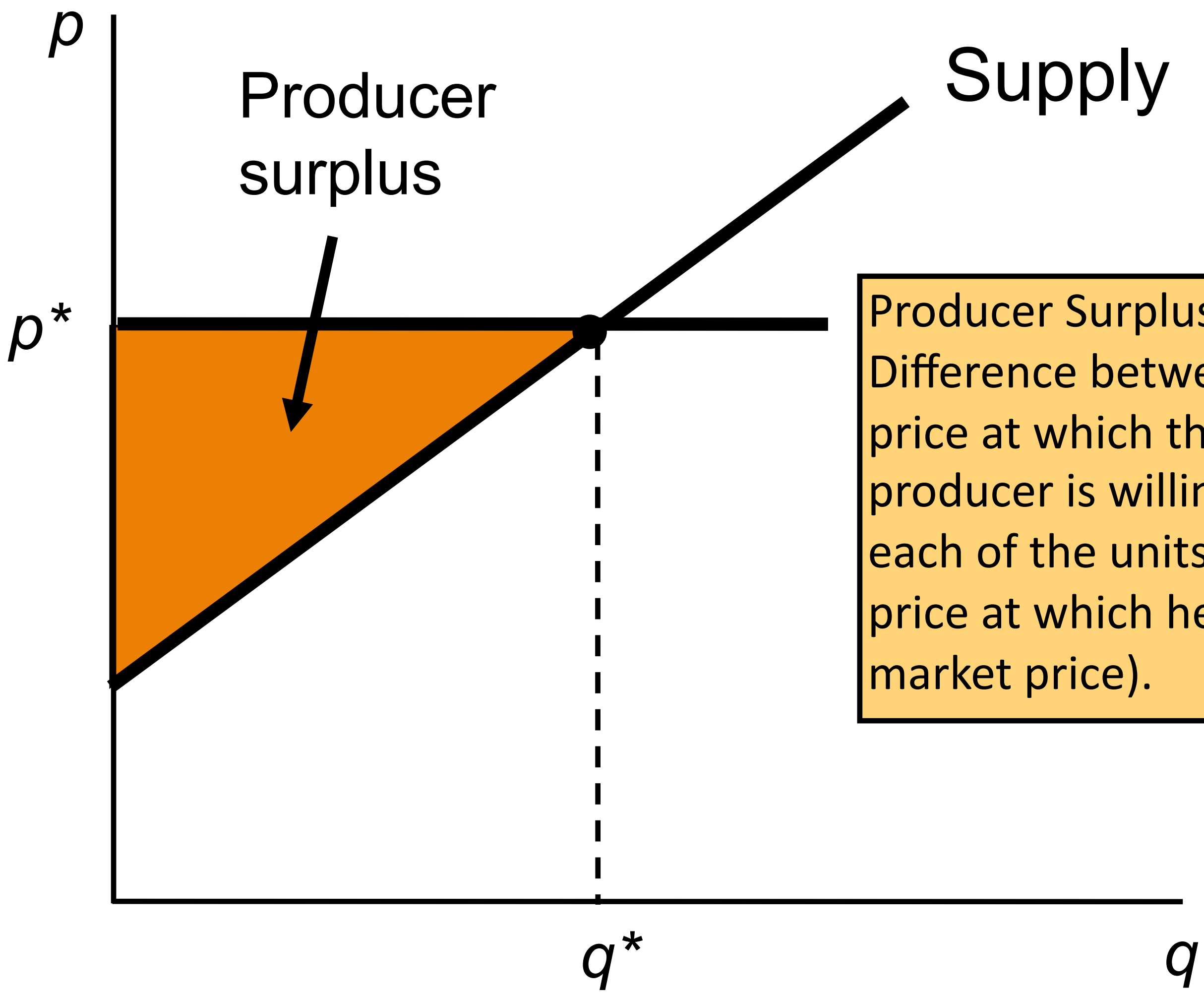
When wage increases, labour demand decreases. The Labour demand function has negative slope.

Labour Demand



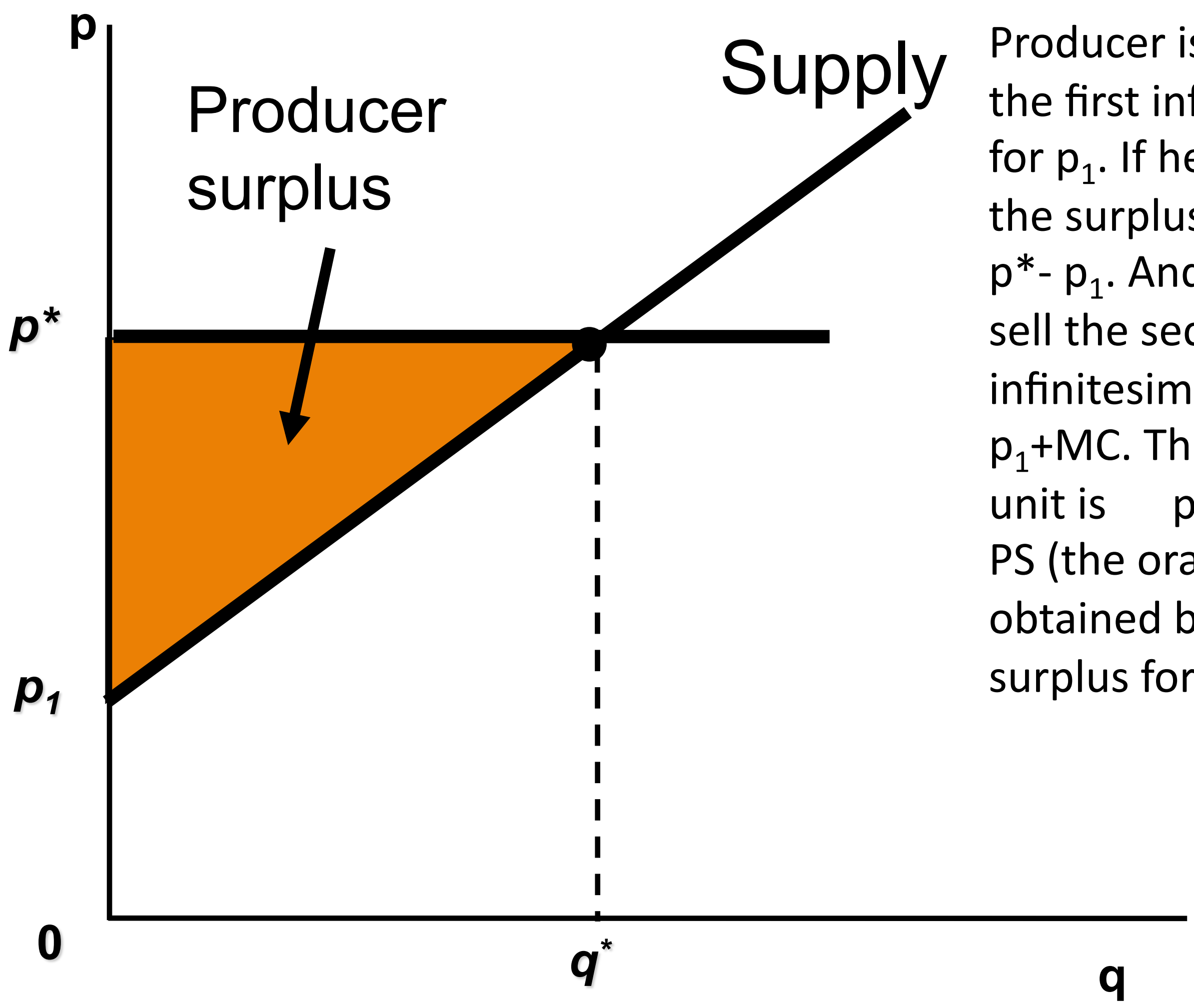
When price of the good decreases, labour demand decreases.

Producer Surplus (PS)



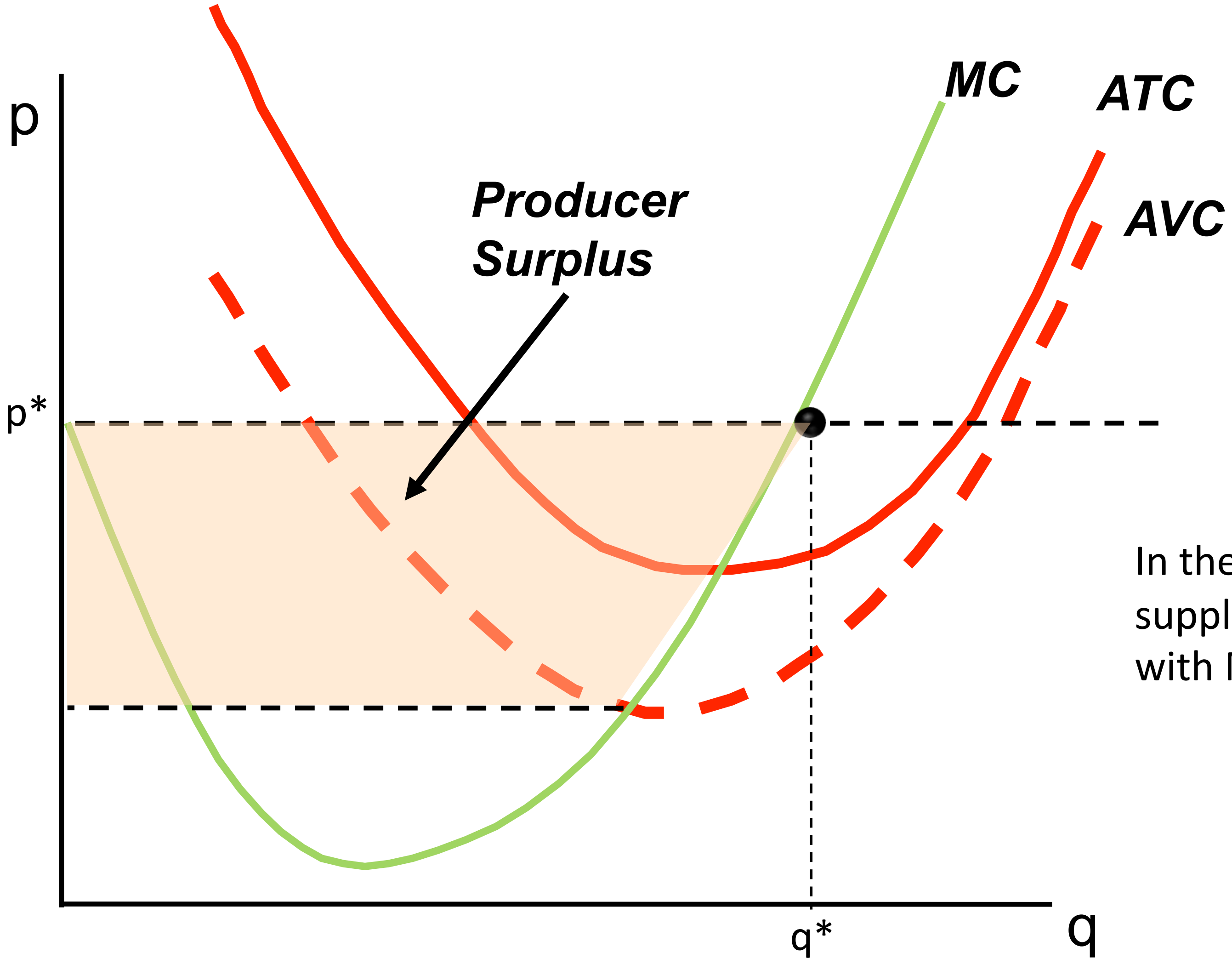
Producer Surplus:
Difference between the price at which the producer is willing to sell each of the units and the price at which he sells (the market price).

Producer Surplus (PS)



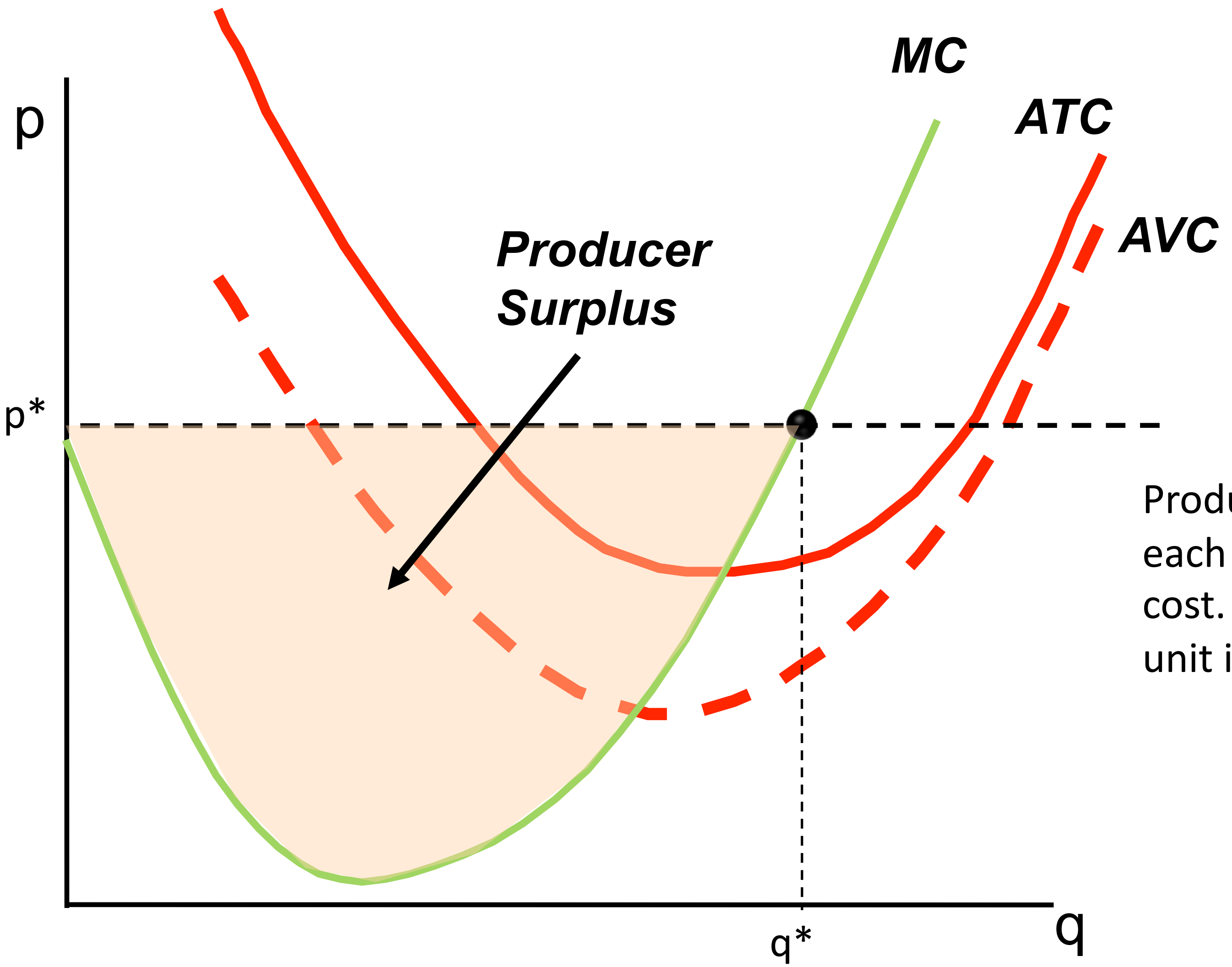
Producer is willing to sell the first infinitesimal unit for p_1. If he sells it for p*, the surplus of that unit is p*- p_1. And he is willing to sell the second infinitesimal unit for p_1+MC. The surplus of that unit is p*- (p_1+MC). The PS (the orange triangle) is obtained by adding the surplus for all units sold.

Producer Surplus - Short Run



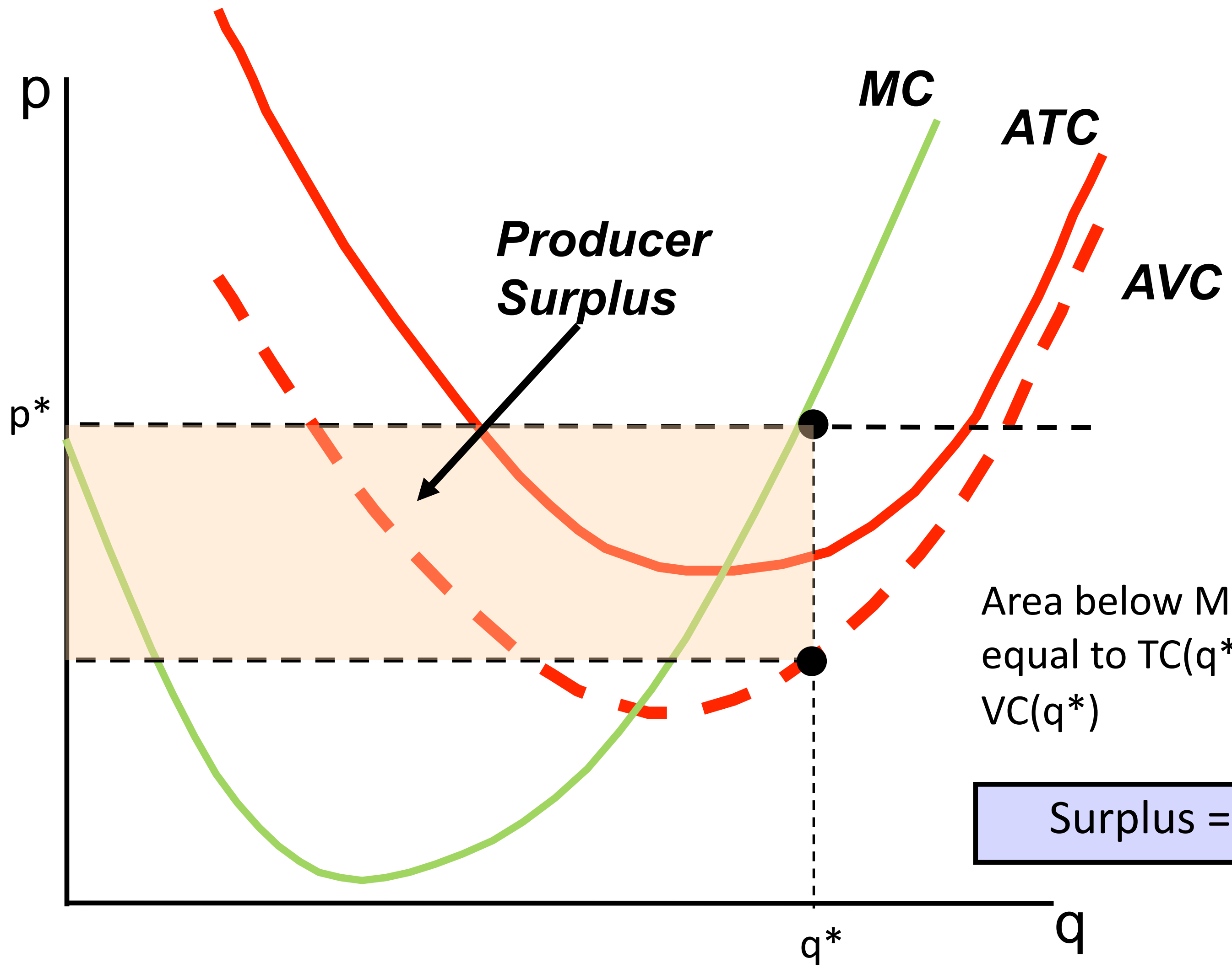
In the short run, supply coincides with MC above AVC

Producer Surplus - Short Run



Producer is willing to sell each unit for its marginal cost. The surplus of each unit is p* - MC.

Producer Surplus - Short Run



Area below MC between 0 and q* is equal to TC(q*) - TC(0) = TC(q*) - CF = VC(q*)

Surplus = p*q* – VC(q*)

Producer Surplus - Short Run

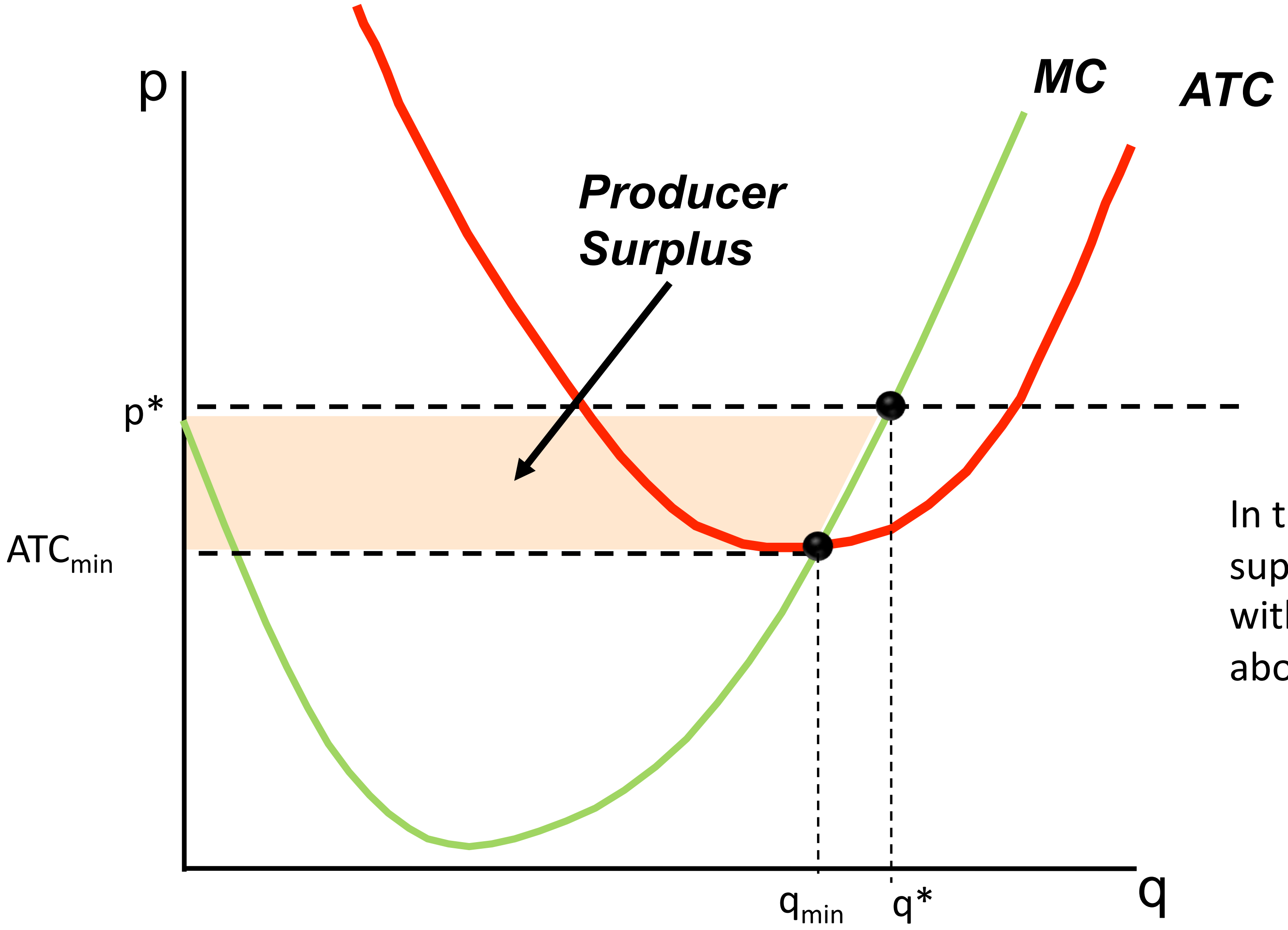
$$PS = p^*q^* - VC(q^*)$$

$$\pi = p^*q^* - VC(q^*) - FC$$

$$\boldsymbol{PS_S = \pi + FC}$$

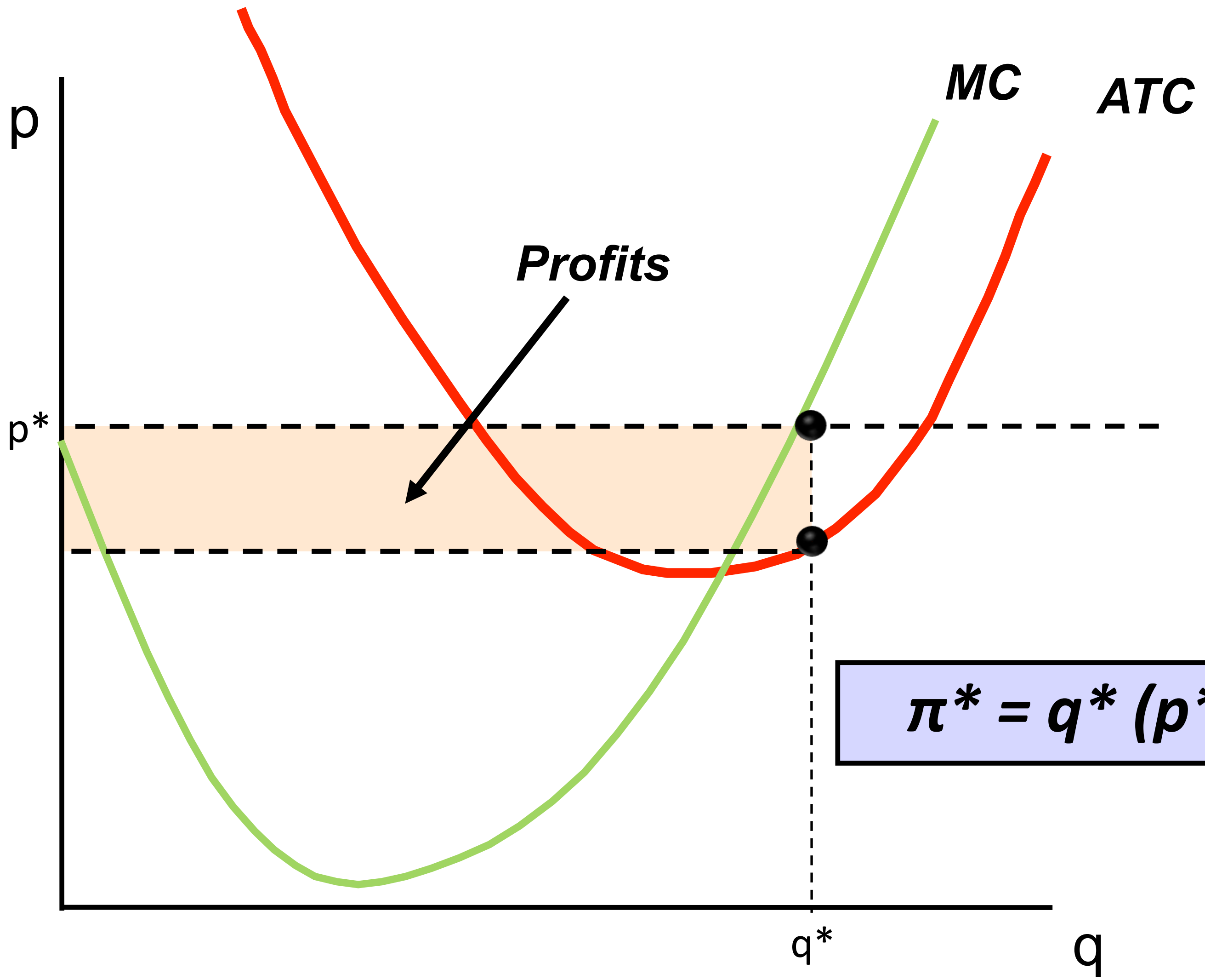
Producer surplus in the short run is greater than profits.

Producer Surplus - Long Run



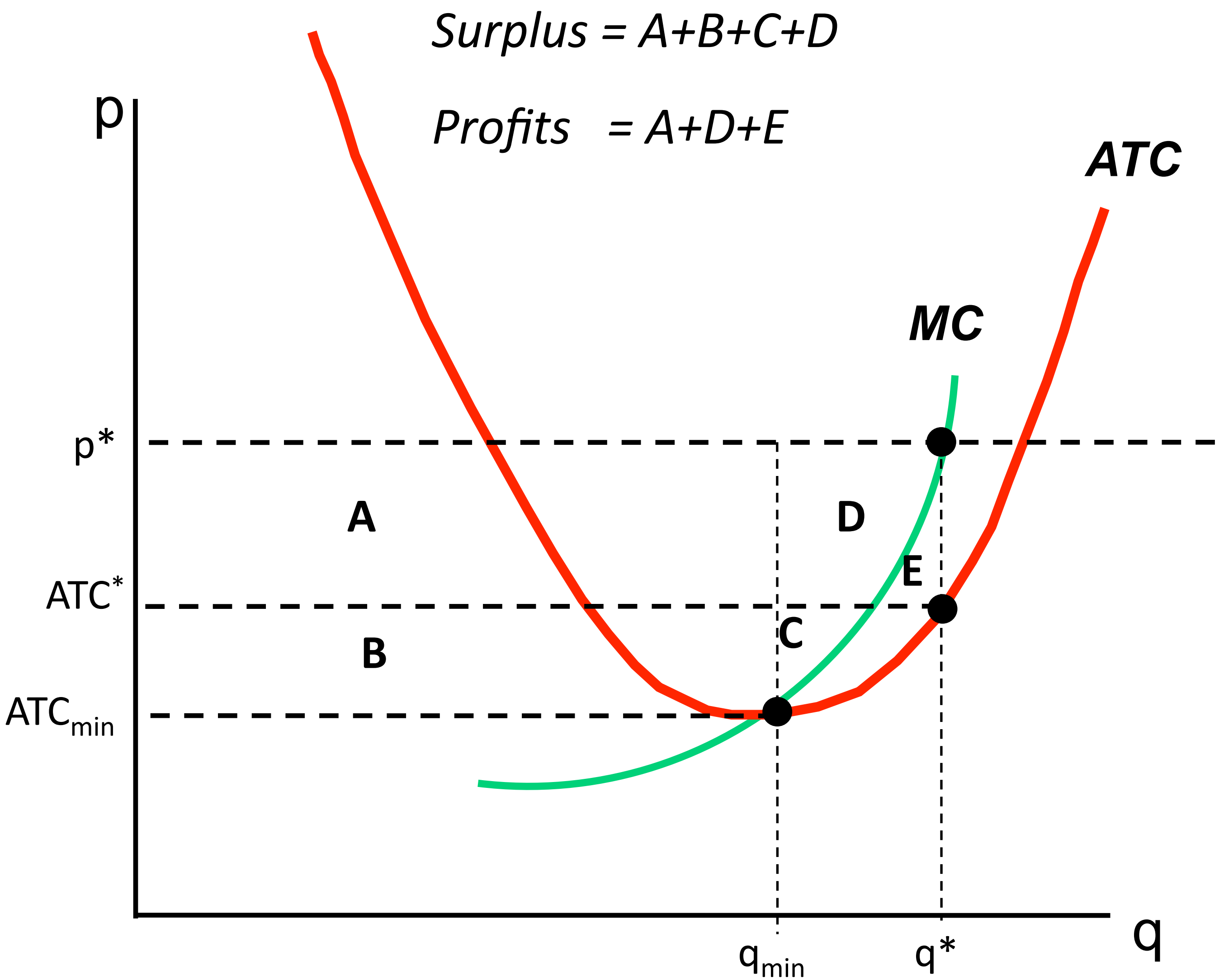
In the long run, supply coincides with increasing MC above ATC

Producer Surplus - Long Run



$$\pi^* = q^* (p^* - ATC(q^*))$$

Producer Surplus - Long Run



Producer Surplus - Long Run

$C+D = (q^* - q_{min})\, p^*$ - area below MC between q^* and q_{min}

$= (q^* - q_{min})\, p^* - (TC(q^*) - TC(q_{min}))$

$= q^*p^* - q_{min}p^* - q^*ATC^* + q_{min}ATC_{min}$

$A+B = q_{min}\, p^* - q_{min}ATC_{min}$

$A+B+C+D = q^*p^* - q^*ATC^* = A+D+E$

$$PS_L = \pi$$

Producer surplus is equal to profits.

Competitive Equilibrium: Long Run

In a long run equilibrium of a competitive market with *free entry* (that is, firms may freely enter or exit the market), and in which technologies are not protected by *patents* or patents have expired (that is, the available technologies may be freely used), firms' profits are ZERO (that is, all factors, including capital, are paid the value of their marginal productivity, and there are no *residual profits*).

Therefore:

- The average cost of every firm is equal to the market price;
- Every firm is producing at its minimum average cost;
- Only the most efficient technologies – that is, the technologies with the lowest minimum average cost are used.

Competitive Equilibrium: Long Run

Hence if there are n different technologies available for the production of a good, and

$$AC^*_i = \min_{Q \geq 0} AC(Q),$$

in the minimum average cost of production using the technology i, then the long run competitive equilibrium price is

$$p^* = \min \{AC^*_1, ..., AC^*_n\}.$$

The level of output in the competitive equilibrium is determined by the demand:

$$Q^* = D(p^*).$$

Finally, the number of firms in the industry is that necessary to produce the output Q* with the efficient technology. If there is a single efficient technology i, and the average cost minimizing level of output is q*, then the number of firms is the long run competitive equilibrium is

$$n^* = [\, Q^*/ q^*].$$

Competitive Equilibrium: Long Run

The dynamics leading to the long run competitive equilibrium is set off by entry and exit of firms motivated by the existence positive and negative profits.

•If some firms have positive profits, new firms will enter the industry with the best technologies available (that is, the most profitable ones). This entry will increase the market supply and lead to a smaller a market prices.

•Smaller prices will eventually cause some firms to have negative profits, and exit the market. These firms are the ones using the less efficient technologies. Exit of these firms will reduce the market supply and increase the market price, leading to high profits for the efficient firms, and to entry of new firms with efficient technologies.

•The market stabilizes when all firms’ have zero profits.