Microeconomics, Final Exam: Test (uc3m, June 1, 2023)

Name: Group:

You have 45 minutes. Mark your answer with an "x." You get 2 points for each correct answer, -0.66 for each incorrect answer, and zero points for each question you do not answer.

1. If an individual's preferences are complete and transitive (axioms A.1 and A.2), but not monotone (axiom A.3) since the individual dislikes consuming good x and enjoys consuming good y, then

☐ cannot be represented by a utility function
☐ her indifference curves can cross
☒ her indifference curves are increasing
☐ her indifference curves have area.

2. If the marginal rate of substitution of a consumer satisfies $MRS(2,2) = 2$, her monetary income is $I = 4$ and prices are $p_x = p_y = 1$, then:

☐ the bundle $(2,2)$ is optimal
☒ she must increase her consumption of x
☐ she must increase her consumption of y
☐ she must increase her consumption of both x and y.

3. A consumer uses electricity and/or gas as perfect substitutes sources of energy to heat her house. She considers heating a normal good, and currently uses gas as energy. Then the substitution and total effects (SE, TE) over the demand of gas of a decrease of the price of gas are:

☐ $TE = 0$, $SE < 0$ ☐ $TE < 0$, $SE = 0$
☐ $TE < 0$, $SE < 0$ ☒ $TE > 0$, $SE = 0$.

4 and 5. An individual's preferences are represented by the utility function $u(x, y) = 2x + y$. In the base period her income is $I = 4$ and prices are $(p_x, p_y) = (2, 2)$. If the prices in the current period are $(p'_x, p'_y) = (3, 1)$, then her true consumer price index (CPI) is

☐ $\frac{2}{3}$ ☐ $\frac{3}{2}$ ☒ 1 ☐ $\frac{4}{3}$,

while her Laspeyres type CPI is

☐ $\frac{2}{3}$ ☒ $\frac{3}{2}$ ☐ 1 ☐ $\frac{4}{3}$.

6 and 7. Daniel's preferences are represented by the Bernoulli utility function $u(x) = \sqrt{x}$. He receives a job offer that pays a wage that depends on the evolution of the economy: if it enters an expansion (A) it pays $x_A = 16$, if it remains stable (B) it pays $x_B = 9$, and if it enters a recession (C) it pays $x_C = 0$. The probability of the scenarios A, B and C are $p_A = 1/2$, $p_B = 1/3$ and $p_C = 1/6$, respectively. If the firm in which Daniel currently works wants to counteroffer, which is the fixed (minimum) wage it must offer?

☐ 7 ☒ 9 ☐ 8 ☐ 6.

If Daniel's current firm offers him the fixed wage that leaves him indifferent between accepting the external offer or not, then Daniel's value of perfect information about the evolution of the economy is

☐ 0 ☒ greater than 3 ☐ 3 ☐ greater than 0, but less than 3.

8. Lolita is a competitive cow that produces milk using oatmeal (O) and hey (H) according to the production function $F(O,H) = \sqrt{2OH}$. Hence, as a milk producer Lolita has:

☒ constant returns to scale ☐ decreasing returns to scale
☐ decreasing marginal cost ☐ diseconomies of scale.

9. A firm produces a good with labor and capital according to the production function $F(L,K) = \min\{2L, K\}$. If the input prices are $w = 2$ and $r = 4$, respectively, the firm's conditional factor demands satisfy:

☐ $L(q) = 2q$ ☐ $K(q) = q^2$
☒ $L(q) = \frac{q}{2}$ ☐ $K(q) = \frac{q}{2}$.

10. A firm produces a good with labor and capital according to the production function $F(L,K) = \sqrt{LK}$. The input prices are $w = r = 1$. In the short run capital is fixed to $\bar{K} = 2$. Hence, the firm's short run cost functions satisfy:

☐ $AC(q) = Q$ ☐ $C(q) = 2 + Q^2$
☒ $MC(Q) = Q$ ☐ $AVC(Q) = \frac{Q}{4}$.

11. If a competitive firm has in the short run economies of scale for production levels $Q < \bar{Q}$ and diseconomies of scale for $Q > \bar{Q}$, then at the price P its supply

☐ is positive ☒ is zero if $P < AVC(\bar{Q})$
☐ is zero if $P < AC(\bar{Q})$ ☐ satisfies $P = MC(Q)$ if $P > AC(\bar{Q})$.

12, 13 and 14. A landlord owns 500 acres of land and monopolizes a rental market for agricultural land in which the demand is $D(P) = \max\{60 - 2p, 0\}$, where p is given in hundreds of euros per acre. The landlord's (opportunity) cost is zero. Hence, in the monopoly equilibrium the landlord rents, in thousand euros, are:

☒ 45 ☐ 30 ☐ 17,5 ☐ 60,

and her Lerner index is

☐ 0 ☒ 1 ☐ $\frac{1}{2}$ ☐ $\frac{2}{3}$.

Further, the increase in the consumer surplus, in thousand euros, that results from the introduction of the maximum price $\bar{p} = 10$ is

☐ 45 ☐ 30 ☒ 17,5 ☐ 60.

15. With respect the monopoly equilibrium without price discrimination, first degree price discrimination produces

☐ a decrease of the total surplus ☐ an increase of consumer surplus
☒ an increase of production ☐ and ambiguous effect on the monopoly's profit.

Microeconomics, Final Exam: Exercises (uc3m, June 1, 2023)

Name: Group:

Exercise 1. A worker's preferences over leisure and consumption are represented by the utility function $u(h, c) = 2\sqrt{h} + c$, where h is the number of months of leisure she enjoys, and c is her annual consumption in *k-euros* (thousands of euros). The worker has 12 months to use as labor or leisure and no other source of income than her labor income. Denoting by w the monthly wage in k-euros, we may normalize the price of consumption to $p_c = 1$.

A. (15 points) Describe the worker's problem, including her budget constraints, and calculate her demand of leisure and consumption, $h(w)$ and $c(w)$, and her supply of labor, $l(w)$.

B. (20 points) Calculate the income and substitution effects of a 25% tax on labor income when the wage is $w = 1$. Also, calculate the compensated variation and verify that it is greater than the tax revenue.

Exercise 2. A firm produces a good using labor L and capital K as inputs, which it hires in competitive markets in which the prices are $w = r = 1$. The firm's production function is $F(L, K) = \sqrt[4]{4L\left(K - \bar{K}\right)}$ if $K \geq \bar{K}$, and $F(L, K) = 0$ if $K < \bar{K}$, where $\bar{K} > 0$ is the minimum capital needed for production. The good's market demand is $D(P) = \max\{60 - P, 0\}$.

A. (10 points) Calculate the firm's conditional inputs demands and its total, average and marginal cost functions.

B. (10 points) Assume that $\bar{K} = 4$ and that the firm monopolizes the market. Calculate monopoly equilibrium and the Lerner index.

C. (10 points) Now assume that there are two types of firms with the given technology, where $\bar{K} = 4$ for type 1 and $\bar{K} = 9$ for type 2. Calculate competitive equilibrium assuming that there are 4 firms of type 1 and 6 firms of type 2.

D. (5 points) Calculate the price and the number of firms in the long run competitive equilibrium with free entry and no patents, when the technologies available to produce the good are those identified in part (C).

Grading Rules:

Exercise 1:

A. Description of the worker's problem (in particular, budget constraints): 2 points. MRS: 2 points. Solution to the system of equations: 5 points. Corner solutions: 3 points. Labor supply: 3 points

B. Substitution effect: 10 points. Income effect: 2 points. Compensated variation: 6 points. Tax revenue and comparison to CV: 2 points.

Exercise 2:

A. MRST: 2 points. System of equations: 2 points. Factor demands: 2 points. Cost functions: 4 points.

B. Equilibrium price and quantity: 8 points. Lerner index: 2 points.

C. Market supply: 8 points. Equilibrium: 2 points.

D. Equilibrium price: 3 points. Equilibrium number of firms: 2 points.

Exercise 1

A. Worker's Problem:

$$\begin{aligned} \max_{h,c}\ u(h,c) &= 2\sqrt{h}+c \\ c+wh &\leq 12w \\ 0 &\leq h \leq 12 \\ c &\geq 0. \end{aligned}$$

The worker's marginal rate of substitution is

$$MRS(h,c) = \frac{\frac{2}{2\sqrt{h}}}{1} = \frac{1}{\sqrt{h}}.$$

An interior solution to the worker's problem solves the system

$$\begin{aligned} \frac{1}{\sqrt{h}} &= w \\ c+wh &= 12w \end{aligned}$$

which solution is

$$h^* = \frac{1}{w^2},\ c^* = 12w - \frac{1}{w}.$$

Since

$$0 \leq \frac{1}{w^2} \leq 12 \Leftrightarrow w \geq \frac{1}{\sqrt{12}},$$

if $w < 1/\sqrt{12}$*, the solution is* $h^* = 12$*,* $c^* = 0$*. Hence, the worker's leisure and consumption demands are*

$$h(w) = \begin{cases} 12 & \textit{if } w < \frac{1}{\sqrt{12}} \\ \frac{1}{w^2} & \textit{if } w \geq \frac{1}{\sqrt{12}} \end{cases},\ c(w) = \begin{cases} 0 & \textit{if } w < \frac{1}{\sqrt{12}} \\ 12w - \frac{1}{w} & \textit{if } w \geq \frac{1}{\sqrt{12}} \end{cases}$$

and her supply of labor is

$$l(w) = \begin{cases} 0 & \textit{if } w < \frac{1}{\sqrt{12}} \\ 12 - \frac{1}{w^2} & \textit{if } w \geq \frac{1}{\sqrt{12}}. \end{cases}$$

B. For $w=1$ *the worker's optimal bundle is*

$$(h\left(1\right), c\left(1\right)) = (1, 11)$$

and her utility is

$$u(1,11) = 2\sqrt{1} + 11 = 13.$$

To calculate the substitution effect we must identify the cheapest bundle that at the effective wage $\hat{w} = 1\left(1 - 1/4\right) = 3/4$ *allows the worker to maintain its utility. To this purpose we need to solve the system of equations:*

$$\begin{aligned} 2\sqrt{h} + c &= 13 \\ \frac{1}{\sqrt{h}} &= \frac{3}{4}, \end{aligned}$$

which solution is

$$\hat{h} = \frac{16}{9},\ \hat{c} = 13 - 2\left(\frac{4}{3}\right) = \frac{31}{3}$$

Thus, the substitution effect is

$$ES = \frac{16}{9} - 1 = \frac{7}{9}.$$

and since

$$TE = h(\frac{3}{4}) - h(1) = \frac{16}{9} - 1 = \frac{7}{9} = SE,$$

the income effect is nil,

$$IE = 0.$$

The compensated variation, CV*, is the subsidy needed for the worker to afford the bundle* $(\hat{h}, \hat{c}) = (16/9, 31/3)$*. If the worker enjoys* $\hat{h}$ *months of leisure, the she works for* $12 - \hat{h}$ *months and her labor income, net of taxes, is*

$$\hat{w}(12 - \hat{h}) = \frac{3}{4}\left(12 - \frac{16}{9}\right) = \frac{23}{3}.$$

Hence

$$CV = \hat{c} - \frac{23}{3} = \frac{8}{3}.$$

The tax revenue is

$$\frac{1}{4}\left(12 - \hat{h}\right) = \frac{1}{4}\left(12 - \frac{16}{9}\right) = \frac{23}{9} < \frac{8}{3} = CV.$$

Exercise 2

A. Obviously, if the firm's wants to produce $Q = 0$*, then the cost minimizing input bundle is* $L = K = 0$ *and its cost is* $C(0) = 0$*. For* $Q > 0$*, the conditional demand of inputs solves the system of equations*

$$\begin{aligned} MRST(L,K) &= \frac{K-\bar{K}}{L} = 1 \\ F(L,K) &= \sqrt[4]{4L\left(K-\bar{K}\right)} = Q, \end{aligned}$$

which solution is

$$L(Q) = \frac{Q^2}{2},\ K(Q) = \frac{Q^2}{2} + \bar{K}.$$

Las total, marginal and average cost functions are

$$\begin{aligned} C(Q) &= L(Q) + K(Q) = \bar{K} + Q^2, \\ MC(Q) &= 2Q \\ AC(Q) &= \frac{\bar{K}}{Q} + Q. \end{aligned}$$

B. For $Q \leq 60$ *inverse demand and the monopoly's revenue are*

$$P(Q) = 60 - Q,\ R(Q) = (60 - Q)\,Q$$

and for $Q > 60$, $P(Q) = R(Q) = 0$*. The output that maximizes the monopoly's profit solves the equation*

$$MR(p) = MC(Q) \Leftrightarrow 60 - 2Q = 2Q.$$

That is,

$$Q_M = \frac{60}{4} = 15.$$

The monopoly equilibrium price is

$$P(Q_M) = 60 - 15 = 45.$$

The Lerner index is

$$L = \frac{P_M - CMa(Q_M)}{P_M} = \frac{45 - 2\,(15)}{45} = \frac{1}{3}.$$

C. The competitive firm supply is obtained by solving the equation

$$P = CMa(Q) \Leftrightarrow P = 2Q$$

for $P \geq AC^*(\bar{K})$ *(minimum average cost, which depends on the parameter* $\bar{K}$*), and* $Q = 0$ *for* $P < CMe^*(\bar{K})$*. To calculate* $CMe^*(\bar{K})$ *we solve the equation*

$$\frac{AC(Q)}{dQ} = \frac{\bar{K}}{Q^2} - 1 = 0 \Leftrightarrow Q^*(\bar{K}) = \sqrt{\bar{K}}.$$

Therefore,

$$AC^*(\bar{K}) = AC(\sqrt{\bar{K}}) = \frac{\bar{K}}{\sqrt{\bar{K}}} + \sqrt{\bar{K}} = 2\sqrt{\bar{K}},$$

that is, the minimum average costs for type 1 and 2 firms are $AC^*(4) = 4$*, and* $AC^*(9) = 6$*. Hence, the supply function for type 1 and 2 firms are*

$$s_1(P) = \begin{cases} 0 & \text{if } P < 4 \\ \dfrac{P}{2} & \text{if } P \geq 6 \end{cases}, \quad s_2(P) = \begin{cases} 0 & \text{if } P < 6 \\ \dfrac{P}{2} & \text{if } P \geq 6 \end{cases},$$

and the market supply is

$$S(P) = 4s_1(P) + 6s_2(P) = \begin{cases} 0 & \text{if } P < 4 \\ 2P & \text{if } 4 \leq P < 6 \\ 5P & \text{if } P \geq 6. \end{cases}$$

The competitive price is the solution to the equation

$$S(P) = D(P) \Leftrightarrow 5P = 60 - P \Leftrightarrow P^* = \frac{60}{6} = 10,$$

and the output of each firm is $Q_i^* = P^*/2 = 5$.

D. The long run competitive equilibrium price is

$$P_L = \min\{CMe^*(4), CMe^*(9)\} = \min\{4, 6\} = 4.$$

Therefore typo 2 firms eventually disappear fro the market, and the firms of type 1 produce $Q^*(4) = 2$ *units. The number of firms of this type in the market* n_L^* *must be enough to serve the demand at the price* $P_L = 4$,

$$D(4) = 60 - 4 = 56.$$

Therefore

$$n_L = \frac{D(4)}{Q^*(4)} = \frac{60 - 4}{2} = 28.$$