1. Master in Industrial Organization and Markets

Microeconomics III (Information Economics)
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Problems on signalling. Answers

Problem 1. In the model of education as a signal, suppose that the utility function of the workers is

$$u_i = w - \frac{2e}{2+a_i},$$

Suppose also that the abilities are $a_g = 5.5$, $a_b = 2$, the reservation utilities are $u_g^R = 3$ and $u_b^R = 0$ and the probabilities of each type are $p(g) = p(b) = 1/2$.

(a) Compute the equilibrium in which the education is not used as a signal.
(b) Compute the most efficient separating equilibrium in which the education is used as a signal. Which of the two equilibria do the workers of type g prefer?

Answer:

1. In this case $e = 0$. The wage will be $w = p(g)a_g + p(b)a_b = \frac{1}{2}(5.5 + 2) = 3.75$. The utilities are $u_g(w, 0) = 3.75 = u_b(w, 0)$. For both agents this utility is greater than the reservation utility, so both will participate.

2. Separating equilibrium. Find e^* such that

$$\begin{aligned} u_g(a_g, e^*) &\geq u_g(a_b, 0) \\ u_b(a_g, e^*) &\leq u_b(a_b, 0) \end{aligned}$$

i.e.

$$\begin{aligned} 5.5 - \frac{2e^*}{2+5.5} &\geq 2 \\ 5.5 - \frac{2e^*}{2+2} &\leq 2 \end{aligned}$$

or

$$7 \leq e^* \leq 13.125$$

The most efficient separating equilibrium corresponds to $e^* = 7$. In this case the utilities are $u_g(a_g, e^*) = 5.5 - \frac{2*7}{7.5} = 5.14 > u_g^R = 3$ and $u_b(a_g, 0) = 2 > u_b^R = 0$.
Note that agent g would be better off under the separating equilibrium.

Problem 2. Consider an economy with three types of workers that differ in their abilities $a_1 = 1,\ a_2 = 2$ and $a_3 = 3$ and their reservation utilities $0.5, 1$ and 2.2. The probability of each type is $1/3$. A perfectly competitive firm considers hiring the workers, but the productivity is private information only known to each worker. Once the workers are hired, the production function of the firm is $y = a_1L_1 + a_2L_2 + a_3L_3$, where L_1 is the amount of work done by a worker of type a_i. The product y is sold at the price of 1.
Suppose that workers can educate themselves. The utility function of the workers is

$$u_i = w - \frac{e}{a_i^2}$$

The company observes the education level e, of each worker.

(a) Compute, if possible, the education levels e_1, e_2 and e_3 such that a worker of type i prefers the education level e_i and the company believes them when he sees the level of education. Note: start by arguing that $e_1 = 0$. Of all the possible levels of education, find those that are better for the consumers. Explain explicitly the beliefs of the company.
(b) Find, if possible, an equilibrium in which $e_1 = e_2 = 0$, and $e_3 > 0$. In this way, the company does not discriminate among the first two types, but distinguish among the first two types and the third one. Compute the equilibrium with the most convenient education level for the workers of type 3. Write explicitly the beliefs of the company. Compare the utilities with those of part (a).

Answer: If each agent chooses a different level of education, and firms beleieve it, the wage each agent will recieve is equal to her productivity a_i

1. Set $e_1 = 0$. The incentive constraints for agent 1 are

$$\begin{aligned} u_1(a_1, 0) &= 1 \geq 2 - e_2 = u_1(a_2, e_2) \\ u_1(a_1, 0) &= 1 \geq 3 - e_3 = u_1(a_3, e3) \end{aligned}$$

for aget 2 we have

$$\begin{aligned} u_2(a_2, e_2) &= 2 - \frac{e_2}{4} \geq 1 = u_2(a_1, e_1) \\ u_2(a_2, e_2) &= 2 - \frac{e_2}{4} \geq 3 - \frac{e_3}{4} = u_2(a_3, e_3) \end{aligned}$$

and for agent 3

$$\begin{aligned} u_3(a_3, e_3) &= 3 - \frac{e_3}{9} \geq 1 = u_3(a_1, e_1) \\ u_3(a_3, e_3) &= 3 - \frac{e_3}{9} \geq 2 - \frac{e_2}{9} = u_3(a_2, e_2) \end{aligned}$$

It is not difficult to check that $e_1 = 0; e_2 = 1$ and $e_3 = 5$ satisfy all those inequalities (and $e_2 = 1$ is the minimum value satisfying it).
The beliefs of the company are

$$\begin{aligned} \mu(a_3 &\mid e \geq 5) = 1; \mu(a_3 \mid e < 5) = 0 \\ \mu(a_2 &\mid e \geq 5) = 0; \mu(a_2 \mid e < 1) = 0; \mu(a_2 \mid 1 \leq e < 5) = 1 \\ \mu(a_1 &\mid e > 1) = 0; \mu(a_1 \mid e < 1) = 1 \end{aligned}$$

2. In this case agents 1 and 2 are pooled togetherthe and the firm offers two wages $w_1 = w_2 = 1.5$ (the average of a_1 and a_2) and $w_3 = 3$. To have an equilibrium of this type it must be true that

$$\begin{aligned} u_1(1.5, 0) &= 1.5 \geq 3 - e_3 = u_1(a_3, e_3) \\ u_2(1.5, 0) &= 1.5 \geq 3 - \frac{e_3}{4} = u_2(a_3, e_3) \\ u_3(a_3, e_3) &= 3 - \frac{e_3}{9} \geq 1.5 = u_3(1.5, 0) \end{aligned}$$

It is easy to see that the minimum e_2 satisfying those consitions is $e_2 = 6$. The beliefs that support this equilibrium are

$$\begin{aligned} \mu(a_3 &\mid e \geq 6) = 1; \mu(a_3 \mid e < 6) = 0 \\ \mu(a_1 &\mid e < 6) = 1/2; \mu(a_2 \mid e < 6) = 1/2 \end{aligned}$$

In this case type 1 agent is better off than in the case above. However, type 2 and type 3 are both worse off