1 Master in Industrial Organization and Markets

Universidad Carlos III
Microeconomics III (Information Economics)
Spring 2012
Problems Moral Hazard. **Answers**

Problem 1. The consumers in an economy own 1 unit of wealth each. They are subject to risk. If a consumer is not careful, she may loose her wealth with probability $p = 0.75$. On the other hand, if a consumer is careful, this probability is 0.25. The utility function of the consumers is $u = \sqrt{x} - e$, where x is wealth and e takes the value 0.1 if the consumer chooses to be careful and 0 otherwise. In the economy, there is perfect competition among the insurance companies which, in addition are risk neutral.

(a) Argue that a policy that insures completely the consumers generates a moral hazard problem.

If the agent buys an actuarially fair insurance contract, assuming that the probability is $p = 0.25$, she doesn't have incentives to be careful (whatever happens she ends up consuming the same) and the true probaility would $p = 0.75$. Notice that even if the insurance policy is based on $p = 0.75$ whenever there is full insurance the agents is not careful.

(b) What would be the price of the above full insurance policy? Would the consumers buy it?

Firms know about this moral hazard problem and will offer (actuarilly fair) full insurance contracts based on the probability $p = 0.75$. In this case the premium is $q = p \times 1 = 0.75$. Her expected utility under this contract is $\sqrt{1 - \frac{3}{4}} = \frac{1}{2}$. If the consumer does not buy insurance and she is carefull her expected utility is $\frac{1}{4}(-0.1) + \frac{3}{4}(\sqrt{1} - 0.1) = 0.65$. Therefore the consumer will not buy insurance

Problem 2. Suppose that in a principal-agent relation there are two possible outcomes, whose values are 10000 and 100 . The agent chooses among two possible efforts, which are not observable by the principal. The distribution of probabilities on outcomes as a function of the efforts is summarized in the following table

		Results	
		10000	100
Efforts	$e = 0$	1/4	3/4
	$e = 2$	3/4	1/4

If the principal incentivates low effort, the maximization problem is

$$\left.\begin{array}{rl} \max_{w_0,w_1} & \frac{1}{4}(10.000 - w_0) + \frac{3}{4}(1.000 - w_1) \\ \text{s.t.} & \frac{1}{4}w_0^{1/4} + \frac{3}{4}w_1^{1/4} \geq 1 \end{array}\right\}$$

The constraint binds and the optimal is attained at salaries $w_0 = w_1 = w$. Hence, $w^{1/4} = 1$ and the solution is $w_0 = w_1 = 1$. The profits for the principal are $\Pi_s(e = 0) = 2574$.

If the principal incentivates high effort, the maximization problem is

$$\left.\begin{array}{rl} \max_{w_0,w_1} & \frac{3}{4}(10.000 - w_0) + \frac{1}{4}(1.000 - w_1) \\ \text{s.t.} & \frac{3}{4}w_0^{1/4} + \frac{1}{4}w_1^{1/4} - 2 \geq 1 \end{array}\right\}$$

The constraint binds and the optimal is attained at salaries $w_0 = w_1 = w$. Hence, $w^{1/4} = 3$ and the solution is $w_0 = w_1 = 81$. The profits for the principal are $\Pi_s(e = 1) = 7444$.

Comparing the profits with low effort ($\Pi_s(e = 0) = 2574$) and high effort ($\Pi_s(e = 1) = 7444$), the principal chooses to incentivate high effort.

The principal is risk-neutral and the agent is risk-averse. The preferences are described by the utility functions

$$B(x, w) = x - w, \text{ and } \quad u(w, e) = w^{1/4} - e$$

where x represents the outcome, w the salary and e is the effort. The level of reservation utility of the agent is $u^R = 1$. Suppose that the principal is a monopolist.

a) Write and solve the problems of the principal when he can observe the effort of the agent. Determine the salaries of the agent and the profits of the principal. Justify what would be the level of effort chosen by the principal.

b) Write and solve the problems of the principal when he cannot observe the effort of the agent. Determine the salaries of the agent and the profits of the principal. What would be the level of effort chosen by the principal?

c) Compare the utilities and profits of the contracts computed in parts (a) and (b). Which situation is preferred by each of the agents? How much would the principal be willing to pay to be able to monitor the effort?

If the principal incentivates low effort, the optimal solution coincides with the symmetric information case. Hence, $w^{1/4} = 1$ and the solution is $w_0 = w_1 = 1$. The profits for the principal are $\Pi_a(e = 0) = 2574$.

If the principal incentivates high effort, the maximization problem is

$$\left.\begin{array}{ll} \max_{w_0,w_1} & \frac{3}{4}(10.000 - w_0) + \frac{1}{4}(1.000 - w_1) \\ \text{s.t.} & \frac{3}{4}w_0^{1/4} + \frac{1}{4}w_1^{1/4} - 2 \geq 1 \\ & \frac{3}{4}w_0^{1/4} + \frac{1}{4}w_1^{1/4} - 2 \geq \frac{1}{4}w_0^{1/4} + \frac{3}{4}w_1^{1/4} \end{array}\right\}$$

The first constraint is the participation (or individual rationality) constraint. The second constraint is called the incentive constraint. Both of them bind and the optimal is attained at salaries w_0 and $w_1 = w$ such that

$$\left.\begin{array}{l} \frac{3}{4}w_0^{1/4} + \frac{1}{4}w_1^{1/4} = 3 \\ \frac{3}{4}w_0^{1/4} + \frac{1}{4}w_1^{1/4} - 2 = \frac{1}{4}w_0^{1/4} + \frac{3}{4}w_1^{1/4} \end{array}\right\}$$

Solving the above system, we obtain that $w_1^{1/4} = 0$, $w_0^{1/4} = 4$. Hence, $w_1 = 0$ and $w_0 = 256$. The profits for the principal are $\Pi_a(e = 1) = 7333$.

Comparing the profits with low effort ($\Pi_a(e = 0) = 2574$) and high effort ($\Pi_a(e = 1) = w$), the principal chooses to incentivate high effort.

The agent (workers) are indifferent and attain their reservation utility in all the situa
Both in the symmetric and asymmetric case, the principal chooses the contract that in
high effort. The principal would be willing to pay up to

$$\Pi_s(e = 1) - \Pi_a(e = 1) = 7444 - 7333 = 111$$

monetary units in order to be able to monitor effort.