# Master in Industrial Organization and Markets. Spring 2012
# Microeconomics III
# Assignment 1: Uncertainty

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## Problem 1

Determine which of the following assertions hold or not. Justify your answers with either an example or a proof : (a) A risk averse agent will never assume any risk. That is he will always choose a sure consumption over a risky one.

**Answer:** No. A risk averse agent might prefer a lottery over a sure consumption if the expected value of the lottery is high enough (and always higher than the value of the sure consumption)

(b) In an economy in which all agents are risk averse, it is not possible to eliminate completely the risk of the agents.

Answer: Wrong. Agents can insurance with each other.

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## Problem 2

Consider an economy with two possible states of nature, with probability 1/5 and 4/5 respectively.
a) Consider the contingent consumption bundles A = (20; 20), B = (30; 15). Which bundle will be chosen by a risk averse agent, depending on his risk attitude? Justify your answer.

**Answer:** A risk averse agent always prefers the expected value of a lottery for sure better than the lottery. The expected value of lottery B is $(1/5)\,30+(4/5)\,15=18$. Since the expected value of this lottery is less than 20, the risk averse agent prefers A better than B

b) Consider the contingent consumption bundles A = (20; 20), B(30; 18). Can we now assure that any agent will prefer B to A, regardless of his risk attitude? Justify your answer.

**Answer:** No. The expected value of lottery B is now 20.4. Depending on the degree of risk aversion the agent might prefer B to A. For example, suppose that the utility function is $u(x)=x^{\alpha}$. If $\alpha$=0.01 (a high degree of risk aversion) $u(20)$=1.0304 and $1/5\,u(30)+4/5\,u(18)=1.0303$. Thus, for this value of $\alpha$ the agents prefers A to B. However, if $\alpha$=0.5 (lower degree of risk aversion) $u(20)$=4.472 and $1/5\,u(30)+4/5\,u(18)=4.489$ and the agent prefers B ot A.

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## Problem 3

Two farmers $A$ and $B$ face the risk of a drought, with probability $q = 1/4$. The drought will reduce their harvest and hence their consumption. The utility function of farmer $i$ ($i = A, B$) in terms of consumption is $v_i(c_i) = (c_i)^{1/2}$, where $c_i$ is $l_i$ if it rains and $s_i$ if it doesn't. Total consumption is 200 if it rains and 100 in case of drought. Both farmers maximize expected utility.

a) What are the Pareto efficient allocations?

**Answer:** The expected utility for agent $i$ is $\mathrm{EU}_i(l_i, c_i) = \frac{3}{4} s_i^{1/2} + \frac{1}{4} l_i^{1/2}$. We saw in class that the Pareto efficient allocations satisfy Marginal Rate Sustitution ($A$) = Marginal Rate Sustituion($B$). The marginal rate of sustition is $\frac{ds_i}{dl_i} \mid_{\mathrm{EU}_i = \mathrm{cte}}$. In our case $\mathrm{MRS}_i = 3 \sqrt{\frac{s_i}{l_i}}$ . Since $s_A + s_B = 100$ and $l_A + l_B = 200$, it is easy to see that the Pareto efficient allocations satisfy $s_A = \frac{1}{2}\, l_A$

Here you have the plot of the indifference curves for the two agents:

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Show[ContourPlot[3/4 √(200 - l) + 1/4 √(100 - s), {l, 2, 200}, {s, 2, 100},
  Contours → 10, ContourShading → None, FrameLabel → Automatic],
 ContourPlot[3/4 √l + 1/4 √s, {l, 2, 200}, {s, 2, 100}, Contours → 10,
  ContourShading → None, FrameLabel → Automatic], PlotRange → All]
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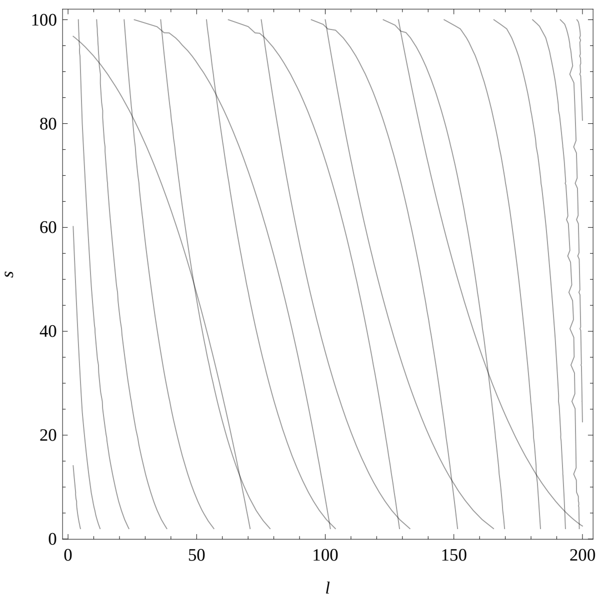



b) Suppose that the farmers can sign ex-ante contingent contracts. Suppose that farmer A would obtain a consumption of 140 if it rains and a consumption of 60 in case of drought. Whereas, farmer B obtains a consumption of 60 if it rains and a consumption of 40 in case of drought. Compute the competitive allocation, assuming both farmers are price takers. Is this allocation efficient? Explain the meaning of those markets.

Answer: Let $p_l$ and $p_s$ be the price of $l$ and $s$, respectively. We know that at equilibrium the ratio $\frac{p_l}{p_s}$ has to be equal to the MRS of the two agents. Thus $\frac{p_l}{p_s} = 3 \sqrt{\frac{s_A}{l_A}}$ .We also know that the equilibrium has to be Pareto efficient, i.e. $s_A = \frac{1}{2}\, l_A$. Therefore $\frac{p_l}{p_s} = 3 \sqrt{\frac{1}{2}}$ , and $p_l = 1$ and $p_s = \frac{\sqrt{2}}{3}$.

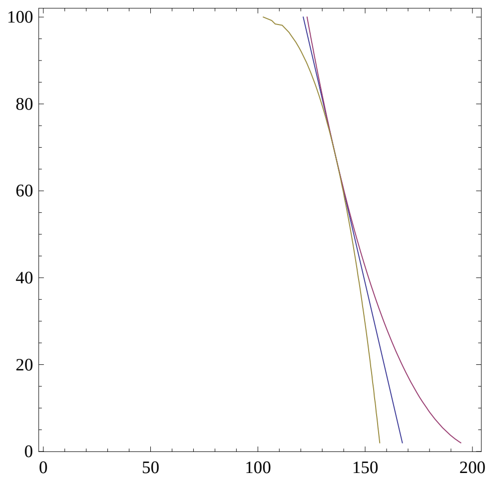
Agent A will consume a bundle that is in his budget contraint, i.e. $l_A + \frac{\sqrt{2}}{3} s_A = 140 + \frac{\sqrt{2}}{3} 60$. Using this equation and the efficiency condition $s_A = \frac{1}{2}\, l_A$we find the optimal consumption for agent A: $l_A^* = 136.185$, and $s_A^* = \frac{136.185}{2}$.

The following plot shows the budget constraint and the indifference curves for the two agents at the equilibrium

$$\texttt{ContourPlot}\Big[\Big\{l + \frac{\sqrt{2}}{3}\, s == 140 + \frac{\sqrt{2}}{3}\, 60,\ \frac{3}{4}\sqrt{136.185} + \frac{1}{4}\sqrt{\frac{136.185}{2}} == \frac{3}{4}\sqrt{l} + \frac{1}{4}\sqrt{s},$$

$$\frac{3}{4}\sqrt{200 - 136.185} + \frac{1}{4}\sqrt{100 - \frac{136.185}{2}} == \frac{3}{4}\sqrt{200 - l} + \frac{1}{4}\sqrt{100 - s}\Big\},$$

$$\{l,\ 2,\ 200\},\ \{s,\ 2,\ 100\}\Big]$$




c) Suppose now that the utility of A with respect to consumption is as before $v_A(c_A) = (c_A)^{1/2}$, but the utility of B with respect consumption is $v_B(c_B) = c_B$ . Without
performing any computations, explain how would be the interior Pareto efficient allocation and how is the risk shared by A and B in these allocations.

**Answer:** Agent B is risk neutral and will get all the risk. Agent A will get full insurance. The MRS of agent B is 3. The effcicient condition $\text{MRS}_A = \text{MRS}_B$ implies that $l_A = s_A$. Here you have the plot of the indifference curves and the Pareto efficient allocations.

```
Show[ContourPlot[3/4 (200 - l) + 1/4 (100 - s), {l, 2, 200}, {s, 2, 100},
  Contours → 10, ContourShading → None, FrameLabel → Automatic],
 ContourPlot[3/4 √l + 1/4 √s, {l, 2, 200}, {s, 2, 100}, Contours → 10,
  ContourShading → None, FrameLabel → Automatic], ContourPlot[3 √(s/l) == 3, {l, 2, 200},
  {s, 2, 100}, ContourShading → None, FrameLabel → Automatic], PlotRange → All]
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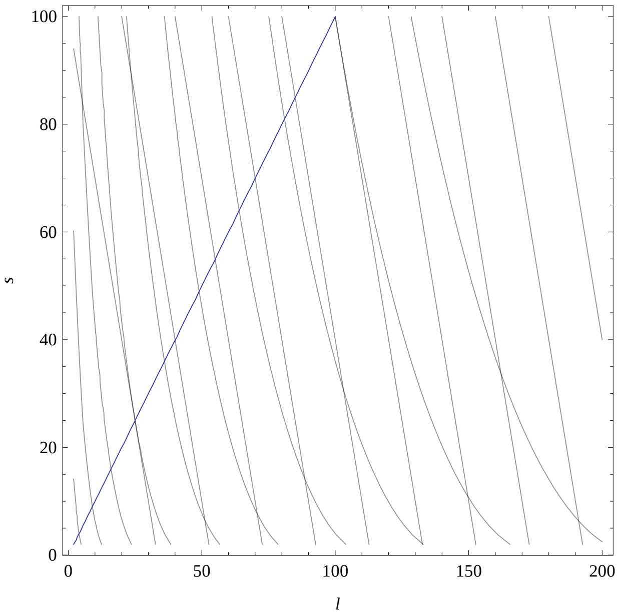



# Problem 4

4. Consider each of the following risk situations faced by a consumer whose utility function is $u = x^{1/2}$.

situation i

| wealth | probability |
|---|---|
| 5 | 3/4 |
| 0 | 1/4 |

situation ii

| wealth | probability |
|---|---|
| 20 | 2/3 |
| 5 | 1/3 |

situation iii

| wealth | probability |
|---|---|
| 10 | 2/4 |
| 5 | 1/4 |
| 0 | 1/4 |

(a) Compute the actuarially fair insurance in each case

(b) Compute the certainty equivalent and the risk premium.

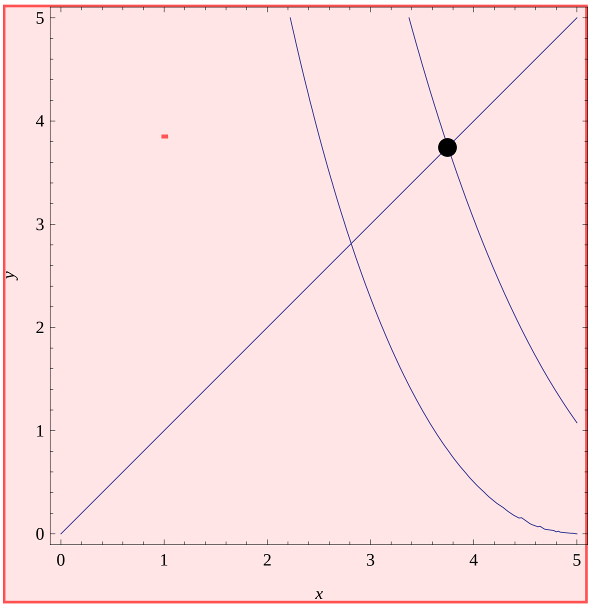
(c) Compute the insurance policy that would impose a monopolist.

(d) Represent graphically your answers in (a) and (c) for each of the cases (i) and (ii).

Answers:
a) The actuarially insurance contracts are given by premium (p) and benefits (b) such that the premium equal to expected benefits. Actuarially fair optimal insurance: Situation i) Insurance= 5; Premium= 5/4; Situation ii) Insurance=15, Premium=15/3, Situation iii) Insurance=(5,10); Premium= 15/4. In this case the insurance is such that if the second outcome happens the agent receives 5 from the insurance company, if the third outcome happens she receives 10. She pays the premium 15/4. Notice that as in the other cases the premium is equal to the expected benefit received from the insurance company.

This is the plot for the first case (the third case can’t be represented using this type of plot since there are three possible outcomes)

$$\text{Show}\left[\text{ContourPlot}\left[\frac{3}{4}\sqrt{x}+\frac{1}{4}\sqrt{y}==\frac{3}{4}\sqrt{5},\ \{x,0,5\},\ \{y,0,5\},\ \text{ContourShading}\to\text{None},\ \text{FrameLabel}\to\text{Automatic}\right],\ \text{ContourPlot}\left[\frac{3}{4}\sqrt{x}+\frac{1}{4}\sqrt{y}==\sqrt{5-(5/4)},\ \{x,0,5\},\ \{y,0,5\},\ \text{ContourShading}\to\text{None},\ \text{FrameLabel}\to\text{Automatic}\right],\ \text{ContourPlot}[x==y,\ \{x,0,5\},\ \{y,0,5\},\ \text{ContourShading}\to\text{None},\ \text{FrameLabel}\to\text{Automatic}]\right]$$




b) Certainty equivalent: the amount *c* such that the agent is indifferent between the lottery and such value *c* for sure (I find it using the Solve command in *Mathematica*)

$$\text{c1} = \text{c} \,/.\, \text{Solve}\left[\sqrt{c} == \frac{3}{4.}5^{1/2},\ c\right][[1]]$$

[illegible]

$$\text{c2} = \text{c} \,/.\, \text{Solve}\left[\sqrt{c} == \frac{2}{3}\sqrt{20}+\frac{1}{3}\sqrt{5.},\ c\right][[1]]$$

[illegible]

$$\text{c3} = \text{c} \,/.\, \text{Solve}\left[\sqrt{c} == \frac{2}{4}\sqrt{10}+\frac{1}{4}\sqrt{5.},\ c\right][[1]]$$

[illegible]

Risk premium

$$5 * \frac{3}{4} - \text{c1}$$

[illegible]

$$20 * \frac{2}{3} + 5 * \frac{1}{3} - \text{c2}$$

1

$$10 * \frac{2}{4} + 5 * \frac{1}{4} - c3$$

[illegible]

c)

Situation i). The monopolist maximizes expect profits $p - \frac{1}{4} b$, where $p$ is the premium and $b$ the benefit, subject to the constraint that the agent should end up with the saem expected utility as in the no-insurance case (I solve it using the command “Maximize” in *Mathematica*. Notice that first I write the objective function and next the constarint):

$$\text{Maximize}\left[\left\{p - \frac{1}{4} b,\ \frac{3}{4}\sqrt{5 - p} + \frac{1}{4}\sqrt{b - p} == \frac{3}{4}\sqrt{5}\right\}, \{p, b\}\right]$$

$$\left\{\frac{15}{16}, \left\{p \to \frac{35}{16}, b \to 5\right\}\right\}$$

In a similar way we find the optimal insurance for the monopolist in situation ii)

$$\text{Maximize}\left[\left\{p - \frac{1}{3} b,\ \frac{2}{3}\sqrt{20 - p} + \frac{1}{3}\sqrt{5 + b - p} == \frac{2}{3}\sqrt{20} + \frac{1}{3}\sqrt{5}\right\}, \{p, b\}\right]$$

$$\left\{\frac{10}{9}, \left\{p \to \frac{5}{9}, b \to 15\right\}\right\}$$

In the third case we have to specify the benefits in the two bad outcomes

$$\text{Maximize}\left[\left\{p - \frac{1}{4} b1 - \frac{1}{4} b2,\ \frac{2}{4}\sqrt{10 - p} + \frac{1}{4}\sqrt{5 + b1 - p} + \frac{1}{4}\sqrt{b2 - p} == \frac{2}{4}\sqrt{10} + \frac{1}{4}\sqrt{5.}\right\}, \{p, b1, b2\}\right]$$

{[illegible], {p → [illegible], b1 → 5, b2 → 10}}

## Problem 5

5. Consider the following lotteries:

- Lottery $x = (x_1, x_2)$: with probability $0 \le p_1 \le 1$ the agent obtains $x_1$ and with probability $p_2 = 1 - p_1$ the agent obtains $x_2$.
- Lottery $y = (y_1, y_2)$: with probability $0 \le p_1 \le 1$ the agent obtains $y_1$ and with probability $p_2 = 1 - p_1$ the agent obtains $y_2$.
  A risk averse agent with utility function $u(x)$ on money is indifferent between the lotteries $x$ and $y$. That is, $p_1 u(x_1) + p_2 u(x_2) = p_1 u(y_1) + p_2 u(y_2) = u(c)$ where $c$ is the certainty equivalent of both lotteries. Prove that if $y_1 \le x_1 \le x_2 \le y_2$ (that is, the variance of lottery $x$ i smaller than the variance of lottery $y$) then:
- $\mathrm{E}(x) \ge \mathrm{E}(y)$.
- The risk premium of $y$ is greater than risk premium of $x$.

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Plot[x, {x, 0, 10}]
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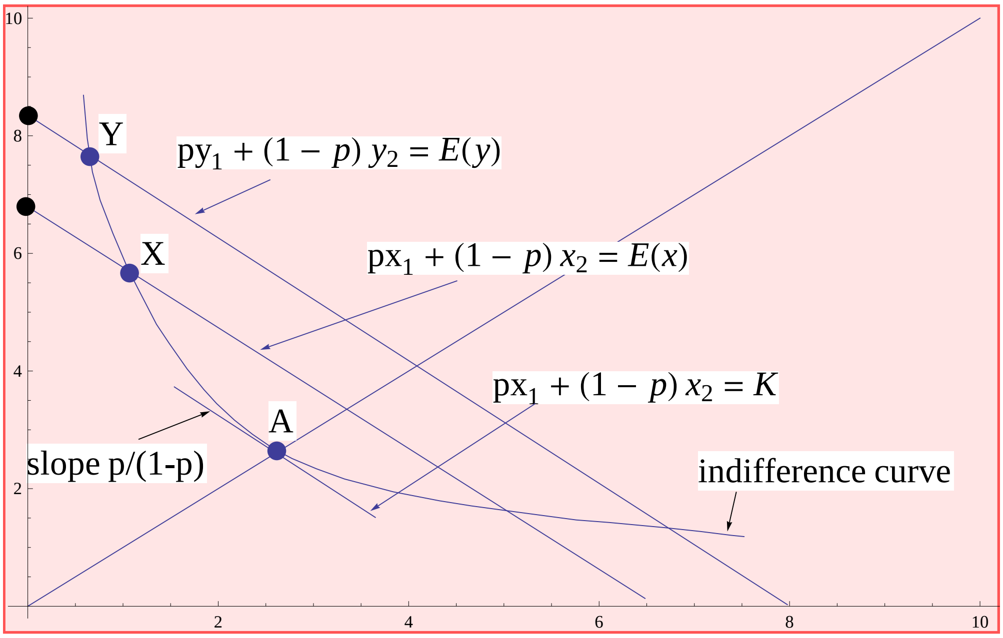
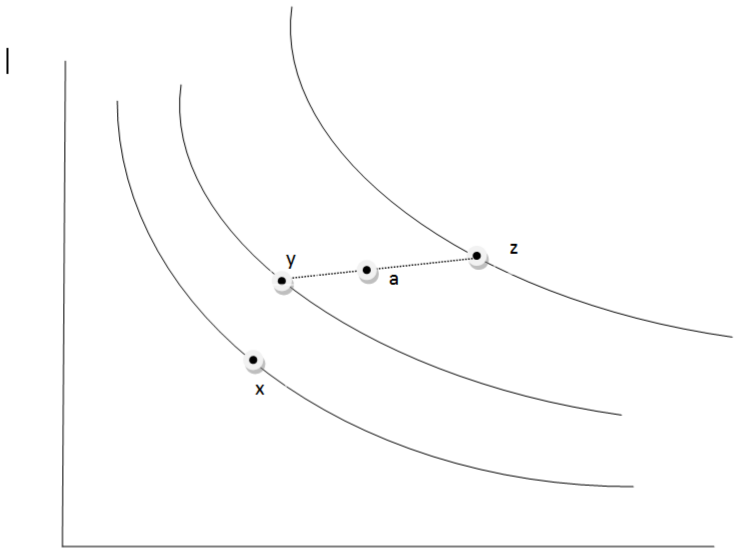



1) Since the agent is risk averse the indifference curve is convex (see figure above)
2) Lotteries X and Y are on the same indifference curve (X and Y in the figure)
3) At the intersection of the indifference curve and the 45º diagonal, point A, the slope of the indifference curve is p/(1-p)
4) The lines defined by $px_1 + (1-p)\,x_2 =$ cte have also slope $p/(1-p)$
5) The line $px_1 + (1-p)\,x_2 = K_1$ that goes through point X, as shown in the figure, has an absolute slope higher than the slope of the indifference curve. The same thing happens with the line $py_1 + (1-p)\,y_2 = K_2$ that goes through Y.
6) It is clear then that $K_2 > K_1$ and since $E(y) = K_2$ and $E(x) = K_1$ we have that $E(y) > E(x)$

Remember that the risk premium is the difference between the certainty equivalent and the expected value. Since the certainty equivalente (A) is the same for both lotteries the risk premium of lottery Y is greater than the risk premium of X.

6. In the context of the above problem, suppose that the agent is risk averse. Fix a lottery $x$. Prove that the set of lotteries $z = (z_1, z_2)$ such that the agent prefers lottery $z$ to lottery $x$ is convex.




If z≻x and $y \succ x$ we have a situation like the one in the figure. Then any convex combination of $z$ and $y$ is a lottery like $a$, and clearly $a \succ x$. It is easy to prove it formally, but this figure is enough for us now.