Master in Industrial Organization and Markets Spring 2012

Microeconomics III

Assignment 1: Uncertainty

The due date for this assignment is Monday April 23.

1. Determine which of the following assertions hold or not. Justify your answers with either an example or a proof:

   (a) A risk averse agent will never assume any risk. That is he will always choose a sure consumption over a risky one.

   (b) In an economy in which all agents are risk averse, it is not possible to eliminate completely the risk of the agents.

2. Consider an economy with two possible states of nature, with probability 1/5 and 4/5 respectively.

   (a) Consider the contingent consumption bundles $A = (20, 20)$, $B = (30, 15)$. Which bundle will be chosen by a risk averse agent, depending on his risk attitude? Justify your answer.

   (b) Consider the contingent consumption bundles $A = (20, 20)$, $B(30, 18)$. Can we now assure that any agent will prefer B to A, regardless of his risk attitude? Justify your answer.

3. Two farmers A and B face the risk of a drought, with probability $q = 1/4$. The drought will reduce their harvest and hence their consumption. The utility function of farmer i ($i = A, B$) in terms of consumption is $v_i(c_i) = (c_i)^{1/2}$, where c_i is l_i if it rains and s_i if it doesn't. Total consumption is 200 if it rains and 100 in case of drought. Both farmers maximize expected utility.

   (a) What are the Pareto efficient allocations?

   (b) Suppose that the farmers can sign ex-ante contingent contracts. Suppose that farmer A would obtain a consumption of 140 if it rains and a consumption of 60 in case o drought. Whereas, farmer B obtains a consumption of 60 if it rains and a consumption of 40 in case o drought. Compute the competitive allocation, assuming both farmers are price takers. Is this allocation efficient? Explain the meaning of those markets.

   (c) Suppose now that the utility of A with respect to consumption is as before $v_A(c_A) = (c_A)^{1/2}$, but the utility of B with respect consumption is $v_B(c_B) = c_B$. Without performing any computations, explain how would be the interior Pareto efficient allocation and how is the risk shared by A and B in these allocations.

4. Consider each of the following risk situations faced by a consumer whose utility function is $u = x^{1/2}$.

situation i

wealth	probability
5	3/4
0	1/4

situation ii

wealth	probability
20	2/3
5	1/3

situation iii

wealth	probability
10	2/4
5	1/4
0	1/4

   (a) Compute the actuarially fair insurance in each case

   (b) Compute the certainty equivalent and the risk premium.

   (c) Compute the insurance policy that would impose a monopolist.

   (d) Represent graphically your answers in (a) and (c) for each of the cases (i) and (ii).

5. Consider the following lotteries:

   - Lottery $x = (x_1, x_2)$: with probability $0 \leq p_1 \leq 1$ the agent obtains x_1 and with probability $p_2 = 1 - p_1$ the agent obtains x_2.
   - Lottery $y = (y_1, y_2)$: with probability $0 \leq p_1 \leq 1$ the agent obtains y_1 and with probability $p_2 = 1 - p_1$ the agent obtains y_2.

     A risk averse agent with utility function $u(x)$ on money is indifferent between the lotteries x and y. That is, $p_1u(x_1) + p_2u(x_2) = p_1u(y_1) + p_2u(y_2) = u(c)$ where c is the certainty equivalent of both lotteries. Prove that if $y_1 \leq x_1 \leq x_2 \leq y_2$ (that is, the variance of lottery x i smaller than the variance of lottery y) then:
   - $\mathrm{E}(x) \geq \mathrm{E}(y)$.
   - The risk premium of y is greater than risk premium of x.

6. In the context of the above problem, suppose that the agent is risk averse. Fix a lottery x. Prove that the set of lotteries $z = (z_1, z_2)$ such that the agent prefers lottery z to lottery x is convex.