May 6, 2019

MATHEMATICS FOR ECONOMICS II (2018-19)

ECONOMICS, LAW–ECONOMICS, INTERNATIONAL STUDIES–ECONOMICS

SHEET 6. DOUBLE INTEGRALS

(1) Compute the following double integrals:

$$a)\int_0^1\int_0^1 xydxdy \qquad b)\int_0^1 dx\int_0^1(2x+y)dy \qquad c)\int_0^1\int_0^1 xy(x+y)dxdy$$

$$d)\iint_{[0,1]\times[1,4]}\sqrt{xy}dxdy \qquad e)\int_0^1\int_0^1(e^x+e^y)dxdy \qquad f)\int_0^1 dy\int_0^1 e^{x+y}dxdy$$

Answers: (a) $\int_0^1\int_0^1 xydxdy = 1/4$

(b) $\int_0^1 dx\int_0^1(2x+y)dy = \frac{3}{2}$

(c) $\int_0^1\int_0^1 xy(x+y)dxdy = 1/3$

(d) $\int_1^4 dy\int_0^1 \sqrt{xy}dx = \frac{28}{9}$

(e) $\int_0^1\int_0^1(e^x+e^y)dxdy = 2(e-1)$

(f) $\int_0^1\int_0^1 e^{x+y}dxdy = (e-1)^2$

(2) Compute the following double integrals:

$$(a)\iint_A xdydx \qquad (b)\iint_B(x+y)dxdy \qquad (c)\iint_C xe^{-x^2/y}dxdy$$

$$(d)\iint_D \frac{1}{(x+y)^2}dydx \qquad (e)\iint_E \frac{x^2}{y^2}dydx \qquad (f)\iint_F \frac{x^2}{y^2}dxdy$$

where

$A = \{(x,y) : x \geq 1, 2x^2-2 \leq y \leq x^2+x\}$,

B the quadrilateral of vertexes $(1,1)$, $(2,2)$, $(3,1)$, $(6,2)$,

$C = \{(x,y) : 0 \leq x \leq \sqrt{y}, 1 \leq y \leq 2\}$,

$D = [3,4]\times[1,2]$,

$E = \{(x,y) : 1 \leq y \leq x, 1 \leq x \leq 2\}$,

$F = \{(x,y) : 1 \leq x \leq y, 1 \leq y \leq 2\}$.

Answers: (a) $\int_1^2\int_{2x^2-2}^{x^2+x} xdydx = \frac{19}{12}$

(b) $\int_1^2\int_y^{3y}(x+y)dxdy = 14$

(c) $\int_1^2\int_0^{\sqrt{y}} xe^{-x^2/y}dxdy = (1-e^{-1})(1-\frac{1}{4})$

(d) $\int_3^4\int_1^2 \frac{1}{(x+y)^2}dydx = \ln\frac{25}{24}$

(e) $\int_1^2\int_1^x \frac{x^2}{y^2}dydx = 2$

(f) $\int_1^2\int_1^y \frac{x^2}{y^2}dxdy = \frac{4}{3}$

(3) (a) Using that $dxdy = \rho\, d\rho d\theta$ in polar coordinates, compute $\iint_A e^{-x^2-y^2}dxdy$, where $A = \{(x,y) : 0 \leq x^2+y^2 \leq R^2\}$.

(b) Compute from (a): $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}dxdy$.

(c) Deduce from (b) the value of $\int_{-\infty}^{\infty} e^{-x^2}dx$.

(d) From (a), using that $\{(x,y) : 0 \leq x^2+y^2 \leq a^2\} \subset [-a,a]^2 \subset \{(x,y) : 0 \leq x^2+y^2 \leq 2a^2\}$, compute approximately $\int_{-a}^{a} e^{-x^2}dx$.

(e) From (d), and supposing that $0 < a < b$, compute approximately $\int_a^b e^{-x^2} dx$.

(f) From (d), and supposing that $a < 0 < b$, compute approximately $\int_a^b e^{-x^2} dx$.

Answers: (a) $\int\int_A e^{-x^2-y^2} dxdy = \pi(1-e^{-R^2})$.

(b) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2} dxdy = \pi$

(c) $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

(d)$\sqrt{\pi(1-e^{-a^2})} < \int_{-a}^{a} e^{-x^2} dx < \sqrt{\pi(1-e^{-2a^2})}$

(e)$\frac{1}{2}\sqrt{\pi(1-e^{-b^2})} - \frac{1}{2}\sqrt{\pi(1-e^{-2a^2})} < \int_a^b e^{-x^2} dx < \frac{1}{2}\sqrt{\pi(1-e^{-2b^2})} - \frac{1}{2}\sqrt{\pi(1-e^{-a^2})}$

(f) $\sqrt{\pi(1-e^{-a^2})} + \sqrt{\pi(1-e^{-b^2})} < \int_a^b e^{-x^2} dx < \frac{1}{2}\sqrt{\pi(1-e^{-2a^2})} + \frac{1}{2}\sqrt{\pi(1-e^{-2b^2})}$

(4) Find the area limited at the right by the circle of radius 2 and at the left by the line $x = 1$.

Answers: $4\pi/3 - \sqrt{3}$.