March 12, 2019

MATHEMATICS FOR ECONOMICS II (2018-19)

ECONOMICS, LAW–ECONOMICS, INTERNATIONAL STUDIES–ECONOMICS

SHEET 3. PRIMITIVES AND INTEGRALS

(1) Find the following integrals:

$$a)\ \int \frac{x^2+x+1}{x\sqrt{x}}dx \qquad b)\ \int xe^{-2x}dx \qquad c)\ \int \sin^{14} x\ \cos x\ dx$$

$$d)\ \int (x+1)(2-x)^{1/3}dx \qquad e)\ \int \frac{x^4}{1+x^5}dx \qquad f)\ \int e^{2x}\ \sin x\ dx$$

$$g)\ \int \frac{1}{3+x^2}dx \qquad h)\ \int x\cos x\ dx \qquad i)\ \int \left(1+\frac{1}{x}\right)^3 \frac{1}{x^2}\ dx$$

$$j)\ \int x\sin ax^2 dx \qquad k)\ \int \frac{x}{\sqrt{16-x^2}}\ dx \qquad l)\ \int \frac{1}{\frac{x^2}{2}-2x+4}\ dx$$

$$m)\ \int \frac{40x}{(x-1)^{40}}\ dx \qquad n)\ \int \frac{x^2+1}{x^3-4x^2+4x}\ dx \qquad o)\ \int \frac{2x+1}{x^3+6x}\ dx$$

$$p)\ \int \frac{x^4}{x^4-1}\ dx \qquad q)\ \int \frac{4x+6}{(x^2+3x+7)^3}dx \qquad r)\ \int \frac{2x-6}{(x-2)^2}dx$$

(2) Evaluate $F'(x)$ in the following cases:

$$a)\ \int_1^x \left(t^2-2t+5\right)\ dt \quad b)\ \int_x^0 t\cos t\, dt \quad c)\ x\left(\int_x^0 t\cos t\, dt\right)$$

(3) Consider the function $F(x)=\displaystyle\int_{-3}^{x}\frac{t^2-4}{3t^2+1}\ dt$.

(a) Find the local maximum points of $F(x)$. Is any of these points a global maximum?

(b) Find the local minimum points of $F(x)$.

Let now $G(x)=\displaystyle\int_{-1}^{x}\frac{t^2-4}{3t^2+1}\ dt$. Does $G(x)$ have a global minimum?

(4) In each case, find the area of the figure bounded by the functions f and g.

$$a)\ f(x)=x^2-4x+3, \quad g(x)=-x^2+2x+3$$

$$b)\ f(x)=(x-1)^3, \quad g(x)=x-1$$

$$c)\ f(x)=x^4-2x^2+1, \quad g(x)=1-x^2$$

(5) Draw the functions $y=2e^{2x}$ and $y=2e^{-2x}$ and find the area bounded by these two functions and the lines $x=-1$, $x=1$.

(6) Find the tangent line to the graph of $f(x)=\sqrt{x}$ at the point $x=4$ and calculate the area of the region enclosed between the graph of f and its tangent line, and the lines $x=0$ and $x=4$.

(7) An asset X pays dividends $D(t)dt$ at instant of time t. The total present value of dividends in the interval $[0,T]$, $T>0$, is

$$V(0)=\int_0^T e^{-rt}D(t)dt,$$

where $r>0$ is the continuous rate of interest of a riskless government bond in the same period. Find $V(0)$ in the following cases.

(a) $D(t)=1$.
(b) $D(t)=2$ up to $\frac{T}{2}$ and $D(t)=0$ in $(\frac{T}{2},T]$.
(c) $D(t)=e^{it}$, where $i>0$.
(d) $D(t)=\sin\frac{\pi t}{T}$ (harder).

(8) Let $f:[0,2]\longrightarrow\mathbb{R}$ be continuous, increasing in $(0,1)$, decreasing in $(1,2)$ and, also, satisfying that: $f(0)=3$, $f(1)=5$ and $f(2)=4$. Between which values can we guarantee that $\int_0^2 f(x)\,dx$ is located?

(9) Let $f:[1,3]\longrightarrow[2,4]$ be increasing, continuous and bijective such that $\int_1^3 f\,dx=5$. Calculate $\int_2^4 f^{-1}(x)\,dx$

(10) Certain company has determined that its marginal cost is $\frac{dC}{dx}=4(1+12x)^{-1/3}$. Find the cost function if $C=100$ when $x=13$.