Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

1

Consider the following system of linear equations in the unknowns (x, y, z, t), where m is a parameter:

$$\begin{cases} x & -2y & -z & +2t & = & 2 \\ 2x & +5y & & -t & = & -1 \\ 3x & +3y & -z & -3t & = & 1 \\ 4x & +y & -2z & +t & = & m \end{cases}$$

(a) (10 points) Study the system according to the values of m.

(b) (10 points) Solve the system for those values of m for which the system admits solutions

2

Consider the symmetric matrix

$$A = \begin{pmatrix} m+1 & 0 & -1 \\ 0 & m & 0 \\ -1 & 0 & m+1 \end{pmatrix},$$

where m is a parameter.

(a) (10 points) The matrix A is diagonalizable for all m. Why? Find the eigenvalues and eigenvectors of A find matrices P regular and D diagonal such that $P^{-1}AP = D$.

(b) (10 points) Classify the quadratic form Q defined by the matrix A.

3

Consider the plane region

$$A = \{(x, y) \in \mathbb{R}^2 \ : \ x \geq 0, x^2 + y^2 \leq 2, y \geq x^2\}.$$

(a) (10 points) Draw A.

(b) (10 points) Calculate the double integral

$$\iint_A x \, dx \, dy.$$

4

Study whether the following improper integrals are convergent or divergent. When convergent, calculate its value.

(a) (10 points)
$$\int_0^1 \frac{dx}{x^2 + x}.$$

(b) (10 points)
$$\int_1^\infty \frac{2x + 1}{(x^2 + x)^3}\, dx.$$

5

(a) (10 points) Study the following limit
$$\lim_{n\to\infty} \frac{p^n}{1+p^n},$$
where $p > 0$ is a parameter. *Hint: $p^n \to 0$ if $0 < p < 1$ and $p^n \to \infty$ if $p > 1$, as $n \to \infty$.*

(b) (10 points) Study the character of the series
$$\sum_{n=1}^{\infty} \frac{p^n}{1+p^n},$$
where $p > 0$ is a parameter. *Hint: use part (a).*