UCC3M

Mathematics for Economics II (final exam)

May 31, 2023

Niu: ________________ Group: ______

Name: ______________________________

Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

1

Consider the following system of linear equations which depend on a and b

$$\begin{cases} x + 2y - 4z + (2b-2)t = 2 \\ \quad - y + bz - 2t = 2 \\ -x + (b-2)y - 5z + 2t = a \end{cases}$$

(a) (15 points) Classify the system in terms of the value of a and b. When the system is consistent (i.e. admits solutions), justify which is the number of parameters needed to describe the solutions.

(b) (5 points) Find the solution or solutions of the system when $b=-3$ and $a=4$.

2

Consider the matrix

$$A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

(a) (10 points) Calculate eigenvalues and eigenvectors of A. Is the matriz diagonalizable?

(b) (10 points) Matrix A can be considered the matrix of a quadratic form Q. Classify Q in the following two cases: (i) in $\mathbb{R}^3$; (ii) restricted to the plane $2x+y=0$.

3

Consider the plane region

$$S = \{(x,y) \in \mathbb{R}^2 \,:\, y \geq x^2 - 3x + 2,\ y \leq 2\}.$$

(a) (10 points) Draw S and calculate its area as a double integral.

(b) (10 points) Claculate

$$\iint_S x\,dxdy,$$

where S is the region considered above.

4

(a) (10 points) Calculate the integral

$$\int_1^\infty \frac{1}{x^3+x}\,dx.$$

Hint: decompose $\frac{1}{x^3+x} = \frac{1}{x(x^2+1)}$ into simple fractions and then use $\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$ and $a \ln b = \ln(b^a)$.

(b) (10 points) The continuous function $f : [0,\infty) \to (0,1]$ satisfies

$$\int_0^x \frac{t}{f(t)}\,dt = \int_0^{x^2} e^{t^2}\,dt$$

for all $x \geq 0$. Find $f(x)$. Hint: use the Fundamental Theorem of Calculus (or the generalization, the Leibniz Rule).

5

(a) (10 points) Consider the sequence $\{x_n\}_{n=1}^\infty$, which satisfies $x_1 = -6$ and $x_{n+1} = \frac{2}{3}x_n + 3$, for all $n \geq 1$.

(i) Prove that the sequence is increasing.

(ii) Prove that the sequence is bounded between -6 and 12, that is, $-6 \leq x_n \leq 12$ for all n

(iii) Deduce that the sequence is convergent and find the limit.