

UC3M
Mathematics for Economics II (final exam)
June 1st, 2022

Niu: _____ Group: _____

Name: _____

Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

Instructions:

- **DURATION OF THE EXAM: 120'.**
- Calculators are **NOT** allowed. **Turn off** your smart phone.
- **DO NOT UNSTAPLE** the exam.
- Please show a valid ID to the professor if required.
- Read the exam carefully. The exam has 5 questions, for a total of 100 points.
- Justify all your answers.

1

(a) (10 points) It is known that

$$\begin{vmatrix} 1 & 0 & 1 & a \\ 2 & -1 & 1 & b \\ 3 & 4 & 2 & c \\ 1 & -1 & 1 & d \end{vmatrix} = 0.$$

Solve for x in the equation

$$\begin{vmatrix} 1 & 0 & 1 & a \\ 2 & -1 & 1 & b \\ 3 & 4 & 2 & c \\ 1 & -1 & 1 & x+d \end{vmatrix} = -10.$$

(b) (10 points) Discuss and solve the linear system

$$\begin{cases} y + z = 1 \\ 2x - y - z = -1 \\ -4x + 5y + 5z = 5 \end{cases}$$

2

Consider the following matrix with parameter $a \neq 0$.

$$A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & a \\ 0 & \frac{3}{a} & 4 \end{pmatrix}.$$

- (a) (10 points) For what values of the parameter a is the matrix A diagonalizable? Justify your answer.
- (b) (10 points) For the values of the parameter a for which the matrix A is diagonalizable, find a matrix P and a diagonal matrix D associated to A . Justify your answer.
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3

- (a) (10 points) Classify the quadratic form $Q(x, y, z) = x^2 + 2y^2 + az^2 + 2xy + 2xz + 4yz$, where $a \in \mathbb{R}$ is a parameter.
- (b) (10 points) Find the value of the double integral

$$\iint_D x e^{xy} dx dy,$$

where $D = [0, 2] \times [0, 1]$.

4

- (a) (10 points) Study the convergence of the improper integral

$$\int_2^{\infty} \frac{1}{x^2} \sin\left(\frac{1}{x}\pi\right) dx$$

and find its value if it results to be convergent.

- (b) (10 points) Let the function $F : [0, \infty) \rightarrow \mathbb{R}$ defined by means of the integral

$$F(t) = \int_0^t x(t-x) dx, \quad \text{for } t \geq 0.$$

Find $F'(t)$.

5

- (a) (10 points) Consider the sequence $\{x_n\}_{n=1}^{\infty}$, which satisfies $x_1 = 10$ and $x_{n+1} = \frac{1}{2}x_n + \frac{1}{5}$, for $n \geq 1$.
Prove that the sequence is convergent and find its limit. Hint: prove that the sequence is monotone decreasing and bounded.
- (b) (10 points) Study the character of the series

$$\sum_{n=1}^{\infty} \frac{\left(\frac{n}{10}\right)^n}{n!}.$$
