UC3M
Mathematics for Economics II (final exam)
May 25th 2021

Niu: _____ Group: _____

Name: _____

Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

Instructions:

- **DURATION OF THE EXAM: 120'**.
- Calculators are **NOT** allowed. **Turn off** your smart phone.
- **DO NOT UNSTAPLE** the exam.
- Please show a valid ID to the professor if required.
- Read the exam carefully. The exam has 5 questions, for a total of 100 points.
- Justify all your answers.

1

Consider the following system of linear equations with parameter $a \in \mathbb{R}$.

$$\left.\begin{array}{rcl} x+z+t & = & 5 \\ x+y+2z+2t & = & 6 \\ x+z+(a+2)t & = & 8 \end{array}\right\}$$

(a) (10 points) Discuss the type of system according to the values of parameter a.

(b) (10 points) Solve the system when $a = 2$. In this case find all solutions that satisfy that $x = 1$.

2

Consider the following matrix with parameters α y β.

$$A = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ \alpha & 1 & -\frac{1}{2} \\ \beta & -\frac{1}{2} & 1 \end{pmatrix}.$$

(a) (10 points) For what values of the parameters α and β is the matrix A diagonalizable? Justify your answer.

(b) (10 points) For the values of the parameters α and β for which the matrix A is diagonalizable, find the matrix P and the diagonal matrix D associated to A. Justify your answer.

3

(a) (10 points) Classify the quadratic form $Q(x, y, z) = -2x^2 - y^2 - 8z^2 + 2xy - 4yz$.

(b) (10 points) Represent the plane set $D = \{(x, y) \in \mathbb{R}^2 : -x^2 \leq y \leq x^2, -1 \leq x \leq 1\}$ and calculate the double integral

$$\iint_D (x^2 - y)dxdy.$$

4

(a) (10 points) Calculate the value of the integral

$$\int_0^6 \frac{2x}{(x^2-4)^{\frac{2}{3}}}\,dx.$$

(b) (10 points) Say whether the following improper integral is convergent or divergent depending on the values of the parameter $\gamma \geq 0$. In the cases where the integral is convergent, calculate its value.

$$\int_0^\infty xe^{-\gamma x}\,dx.$$

5

(a) (10 points) Calculate

$$\lim_{n\to\infty} e^{(\sqrt{n^2+\pi n+2}-n)}.$$

(b) (10 points) Study the character of the series

$$\sum_{n=1}^{\infty} n^{-p}p^n, \quad \text{where } p > 0.$$