UC3M
Mathematics for Economics II (retake)
June 18th 2021

Niu: ________________ Group: ______

Name: ________________________________

Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

Instructions:

- **DURATION OF THE EXAM: 120'**.
- Calculators are **NOT** allowed. **Turn off** your smart phone.
- **DO NOT UNSTAPLE** the exam.
- Please show a valid ID to the professor if required.
- Read the exam carefully. The exam has 5 questions, for a total of 100 points.
- Justify all your answers.

$\boxed{1}$

Consider the following system of linear equations with parameters $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$$\left\{ \begin{array}{rcl} ax + ay + z & = & 1 \\ x + y + az & = & b \\ x + ay + z & = & a \end{array} \right.$$

(a) (15 points) Discuss the type of system according to the values of parameters a and b.

(b) (5 points) Solve the system when $a = -1$.

$\boxed{2}$

Consider the following matrix with real parameters $\alpha \neq 0$ and β.

$$A = \begin{pmatrix} \beta & 0 & 0 \\ 1 & 0 & -\alpha \\ 0 & -\alpha & 0 \end{pmatrix}$$

(a) (10 points) For what values of the parameters $\alpha \neq 0$ and β is the matrix A diagonalizable? Justify your answer.

(b) (10 points) For the values of the parameters $\alpha \neq 0$ and β for which the matrix A is diagonalizable, find the matrix P and the diagonal matrix D associated to A. Justify your answer.

Hint: The condition $\alpha \neq 0$ is important for reducing the number of cases!

$\boxed{3}$

(a) (10 points) Classify the quadratic form $Q(x, y, z) = 4x^2 + y^2 + z^2 - 2cxy - 4xz$, where c is a real parameter.

(b) (10 points) Draw the planar set $D = \{(x, y) \in \mathbb{R}^2 \, : \, -\sqrt{9 - x^2} \leq y \leq 1, 0 \leq x \leq 3\}$ and calculate the double integral

$$\iint_D x dx\, dy.$$

$\boxed{4}$

(a) (10 points) Discuss the character of the improper integral

$$\int_1^e \frac{1}{x \ln x} \, dx.$$

In case that the integral is convergent, find its value.

(b) (10 points) Knowing that $\lim_{x\to\infty} \frac{x}{e^x} = 0$, compute the value of the integral

$$\int_2^\infty (x-1)e^{-x} \, dx.$$

$\boxed{5}$

(a) (10 points) It is known that the sequence of real numbers (x_n) satisfies the following properties:

- $x_1 = \frac{1}{2}$;
- $x_{n+1} = \sqrt{2x_n + 3}$;
- It is convergent to the real number L, that is, $\lim_{n\to\infty} x_n = L$.

Find the value of L.

(b) (10 points) Study the character of the series

$$\sum_{n=2}^{\infty}(a^{\frac{1}{2(n-1)}} - a^{\frac{1}{2n}}), \quad \text{where } a > 0.$$

When the series converges, find its value.

Hint: Note that it is a telescoping series.