UC3M
Mathematics for Economics II (final exam)
May 27th 2019

Niu: ______________ Group: ______

Name: ______________________________

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	10	10	60
Score:							

1

Consider the linear system with parameters $\lambda \neq 0$ and $\mu \in \mathbb{R}$.

$$\begin{cases} x + \lambda y + \mu z &= 1 \\ x + \lambda\mu y + z &= \lambda \\ \mu x + \lambda y + z &= 1 \end{cases}$$

(a) (7 points) Discuss the system (recall that $\lambda \neq 0$).
(b) (3 points) Solve the system for the values $\lambda = \mu = -2$.

2

Consider the matrix with parameters a and b

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & b & a \\ 0 & 0 & b \end{pmatrix}.$$

(a) (5 points) For what values of the parameters a and b is the matrix A diagonalizable? Justify your answer.
(b) (5 points) For the values $a = 0$ and $b = 2$, find the matrix P and the diagonal matrix D associated to A.

3

(a) (3 points) Represent the set $B = \{(x, y) \in \mathbb{R}^2 \,:\, x^2 \leq y \leq x + 2\}$. Calculate the minimum and maximum value of x if $(x, y) \in B$.
(b) (7 points) Calculate the double integral

$$\iint_B (-1 + y)dxdy.$$

4

(a) (4 points) Given $a > 0$, calculate

$$\int_0^a x\sqrt{a^2 - x^2}dx.$$

(b) (6 points) Given the function $f(x) = \dfrac{2}{x^2 - 4x + 3}$, calculate its vertical asymptotes. Study whether the following improper integral converges

$$\int_2^4 \frac{2}{x^2 - 4x + 3}dx.$$

5

(a) (5 points) The sequence $\{x_n\}_{n=1}^{\infty}$ satisfies $x_{n+1} = \frac{1}{2}x_n^2 + \frac{1}{4}\sqrt{n}$, for all $n = 1, 2, \ldots$ and $x_1 = 1$. Study if the sequence is convergent.
(b) (5 points) Calculate $\displaystyle\lim_{n\to\infty} \left(\frac{n}{n+2}\right)^{2n}$.

6

Let the series $\sum_{n=1}^{\infty} a_n$, where either $a_n = \frac{1}{2^n}$ or $a_n = \frac{1}{3^n}$, depending on the index n.

(a) (3 points) Prove that the series is convergent.
(b) (3 points) Calculate the biggest A and the smallest B such that

$$A \leq \sum_{n=1}^{\infty} a_n \leq B.$$

(c) (4 points) Let us consider now the series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$, where either $a_n = \frac{1}{2^n}$ or $a_n = \frac{1}{3^n}$, depending on the index n. Calculate the smallest C such that

$$\sum_{n=1}^{\infty}(-1)^{n+1}a_n \leq C.$$
