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|-----------|----|----|----|----|----|----|-------|
| Question: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
| Points: | 10 | 10 | 10 | 10 | 10 | 10 | 60 |
| Score: | | | | | | | |

1

Consider the linear system with parameter λ .

$$\begin{cases} x + y + \lambda z = \frac{1}{\lambda} \\ x + \lambda y + z = \lambda \\ \lambda x + y + z = 1 \end{cases}$$

- (a) (7 points) Discuss the system.
(b) (3 points) Solve the system for the value $\lambda = 2$.

2

Consider the matrix with parameters a and b

$$A = \begin{pmatrix} 4 & 2 & b \\ 1 & 3 & 2 \\ 0 & 0 & a \end{pmatrix}.$$

- (a) (5 points) For what values of the parameters a and b is the matrix A diagonalizable? Justify your answer.
(b) (5 points) For the values $a = 0$ and $b = 0$, find the matrix P and the diagonal matrix D associated to A .

3

- (a) (3 points) Represent the set $B = \{(x, y) \in \mathbb{R}^2 : y \leq x + 1, y \leq 1 - x, y \geq x^2 - 1\}$.
(b) (7 points) Calculate the double integral

$$\iint_B x^2 dx dy.$$

4

- (a) (6 points) Given $a > 0$, classify the quadratic form $Q(x, y, z) = ax^2 + 4ay^2 + 4az^2 + 4xy$.
(b) (4 points) Study whether the following improper integral converges

$$\int_4^\infty \frac{2}{x^2 - 4x + 3} dx.$$

If it converges, compute its value

5

Given the sequence $x_{n+1} = x_n^2 + \frac{1}{n}$, for all $n = 1, 2, \dots$ and $x_1 = 1$, answer the following questions.

- (a) (5 points) Study whether the sequence is monotone.
(b) (5 points) Study whether the sequence is convergent, and in this case calculate its limit as $n \rightarrow \infty$.

6

Let the series $\sum_{n=1}^\infty (-1)^{n+1} \frac{n}{n^2 + 1}$.

- (a) (5 points) Study if the series is absolutely convergent.
(b) (5 points) Study if the series is convergent and in this case, give an estimate of the maximum error committed when the infinite series is approximated by the sum of the first three terms.