Department of Economics
Mathematics II. Final Exam. September 2008

Last Name:		**Name:**
DNI:	**Degree:**	**Group:**

IMPORTANT

- **DURATION OF THE EXAM: 2h. 30min.**
- Calculators are **NOT** allowed.
- **Scrap paper:** You may use the last two pages of this exam and the space behind this page. **Do NOT UNSTAPLE** the exam.
- You must show a valid ID to the professor.
- Read the exam carefully. Each part of the exam counts $0'5$ points. Please, check that there are 14 pages in this booklet

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
Total	

(1) Consider the following system of linear equations,

$$\begin{cases} ax - 2y + z &= 1 \\ x + ay + 2z &= a \\ x + z &= 1 \end{cases}$$

where $a \in \mathbb{R}$ is a parameter.
 (a) Classify the above system according to the values of the parameter a.
 (b) Solve the above system for the value of a for which the system is underdetermined. How many parameters are needed to describe the solution?

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(2) Consider the matrix

$$A = \begin{pmatrix} 1 & -3 & 1 \\ 0 & -1 & 0 \\ 0 & -2 & a \end{pmatrix}$$

 (a) Find the characteristic polynomial and the eigenvalues of the matrix A.
 (b) For what values of the parameter a, is the matrix diagonalizable?
 (c) For the value $a = 0$, find the corresponding diagonal matrix and the matrix change of basis.

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(3) Given the linear map $f : \mathbb{R}^3 \to \mathbb{R}^4$,

$$f(x, y, z) = (x + y - z, 2x + y + z, 3x + 2y, -x - 2z)$$

 (a) Write down the matrix of f (with respect to the canonical basis of $\mathbb{R}^3$ and $\mathbb{R}^4$). Compute the dimensions of the kernel and the image of f.
 (b) Write a homogeneous system of equations that determines the kernel of f and a homogeneous system of equations that determines the image of f. What is the minimum number of equations necessary to describe each of these systems?
 (c) Find a basis of the image of f and a basis of the kernel of f.

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(4) Consider the set $A = \{(x, y) \in \mathbb{R}^2 : x, y > 0; \ln(xy) \geq 0\}$.
 (a) Draw the set A, its boundary and its interior. Discuss whether the set A is open, closed, bounded, compact and/or convex. You must explain your answer.
 (b) Consider the function $f(x, y) = x + 2y$. Is it possible to use Weierstrass' Theorem to determine whether the function attains a maximum and a minimum on A? Draw the level curves of f, indicating the direction in which the function grows.
 (c) Using the level curves of f, find graphically (i.e. without using the first order conditions) if f attains a maximum and/or a minimum on A.

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(5) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x, y) = \begin{cases} \frac{xye^x}{x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

 (a) Study if the function f is continuous at the point $(0, 0)$. Study at which points of $\mathbb{R}^2$ the function f is continuous.
 (b) Compute the partial derivatives of f at the point $(0, 0)$, if they exist. Is the function f differentiable at the point $(0, 0)$?

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(6) Classify the quadratic form

$$Q(x, y, z) = 3x^2 + 3y^2 + 57z^2 - 6axy$$

depending on the values of the parameter a.

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(7) Consider the function

$$f(x, y) = x^2 + y^2 + x^2y + 4$$

 (a) Compute the critical points of f and classify them.

(b) Determine the convex and open sets in $\mathbb{R}^2$ where the function f is concave, and the convex and open sets in $\mathbb{R}^2$ where the function f is convex.
(c) Determine if f attains any global extreme points on the set $\mathbb{R}^2$.

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(8) Consider the function

$$f(x, y, z) = 4x + 2y + z$$

and the set

$$A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 + z^2 = 21\}$$

(a) Find the Lagrange equations that determine the extreme points of f on the set A.
(b) Determine the points that satisfy the Lagrange equations.
(c) Using the second order conditions, classify the critical the points that satisfy the Lagrange equations.
