University Carlos III of Madrid

Department of Economics
Mathematics II. Final Exam. June 2006.

IMPORTANT:

- **DURATION OF THE EXAM: 2h. 30min.**
- Calculators are **NOT** allowed.
- **Scrap paper:** You may use the last two pages of this exam as scrap paper. These last two pages will not be graded. **Do NOT** unstaple the exam.
- You must show a valid ID to the professor.
- Each part of the exam counts $0'5$ points.

Last Name: **Name:**

DNI: **Group:**

(1) Consider the following system of linear equations,

$$\left.\begin{array}{ccccc} x & +2y & & -t & = & b \\ x & & +az & +t & = & b \\ 3x & +6y & +az & -3t & = & 3 \end{array}\right\}$$

donde $a, b \in \mathbb{R}$ son parámetros.

(a) Classify the system according to the values of the parameters a, b.

(b) Solve the above system for the values of $a = 0$ y $b = 1$.

(2) Given the matrix

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 3 \end{pmatrix}$$

(a) Find the characteristic polynomial and check that the eigenvalues are $\lambda_1 = 1$ y $\lambda_2 = 2$. ¿which one of them has multiplicity?

(b) Determine the corresponding diagonal matrix and the matrix change of basis.

(c) Compute A^5. (It is enough to write the result as a product of three matrices. You do not have to perform additional computations)

---

(3) Given the linear map $f : \mathbb{R}^2 \to \mathbb{R}^5$,

$$f(x, y) = (x + y, 2x + 2y, x + y, 3x + 3y, 4x + 4y)$$

(a) Compute the dimensions of the kernel and the image and a set of equations for these subspaces.
(b) Find a basis of the image of f and a basis of the kernel of f.

---

(4) Given the set

$$S = \{(x,y) \in \mathbb{R}^2 \text{ tales que } 0 \leq y \leq x^2, \quad 0 \leq x \leq 1\}$$

(a) Draw the set S, its boundary and interior and discuss whether the set S is open, closed, bounded, compact and/or convex. You must explain your answer.

(b) Show that the function

$$f(x,y) = \frac{1}{x+y-3}$$

defined on the set S, has a maximum and a minimum on the set S.

(c) Draw the level curves of $f(x,y)$ and determine where the maxima and the minima of f on S.

---

(5) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x,y) = \begin{cases} \frac{x^2}{\sqrt{x^2+y^2}} & \text{si } (x,y) \neq (0,0), \\ 0 & \text{si } (x,y) = (0,0). \end{cases}$$

(a) Study if the function f is continuous at the point $(0,0)$.
(b) Compute the partial derivatives of f at the point $(0,0)$.
(c) Is the function f differentiable at the point $(0,0)$.

---

(6) Consider the quadratic form $Q(x, y, z) = 2x^2 + 2y^2 + 2z^2 - 2axy + 2axz$,

(a) Find the matrix associated to Q.

(b) Study the sign of Q according to the values of the parameter a.

---

(7) Considera la función $f(x,y) = -2xy^2 + 4x$. Se pide,

(a) Obtener los puntos críticos de f.

(b) Clasificar los puntos críticos de f, obtenidos en el apartado anterior.

(8) Consider the function

$$f(x, y, z) = -\frac{x^3}{3} - 2y^2 - 4z^2$$

and the set

$$A = \left\{ (x, y, z) : \frac{x^2}{2} + y + z = \frac{1}{8} \right\}.$$

(a) Find the Lagrange equations that determine the extreme points of f in the set A.
(b) Determine the points that satisfy the Lagrange equations.
(c) One of the points computed in the previous part is a maximum of f on A (it is not necessary to prove this). Determine which of the points computed in part (b)correspond to a a maximum of f on A.

SCRAP PAPER (WILL NOT BE GRADED)

SCRAP PAPER (WILL NOT BE GRADED)

SCRAP PAPER (WILL NOT BE GRADED)

SCRAP PAPER (WILL NOT BE GRADED)