University Carlos III
Department of Economics
Mathematics II. Final Exam. May 2009

Last Name:		**Name:**
ID number:	**Degree:**	**Group:**

IMPORTANT

- **DURATION OF THE EXAM: 2h**
- Calculators are **NOT** allowed.
- **Scrap paper:** You may use the last two pages of this exam and the space behind this page.
- **Do NOT UNSTAPLE** the exam.
- You must show a valid ID to the professor.
- Read the exam carefully. Each part of the exam counts 1 point. Please, check that there are 10 pages in this booklet

Problem	Points
1	
2	
3	
4	
5	
Total	

(1) Consider the set $A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 2, \quad y \ge x^2\}$.

(a) Draw set A, its boundary and interior set. Discuss whether A is open, closed, bounded, compact and/or convex. Clearly explain your answer.

(b) Consider the function

$$f(x,y) = \begin{cases} \frac{1}{x^2+(y-1)^2} & \text{if } (x,y) \neq (0,1), \\ 0 & \text{if } (x,y) = (0,1). \end{cases}$$

Determine if this function achieves a maximum in A. Does it achieve a minimum?

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(2) Consider the following system of equations

$$\begin{cases} y^2 + z^2 - x^2 & = & -4 \\ e^{y-1} + x - z^2 & = & 0 \end{cases}$$

(a) Show that the above system of equations determines y and z as differentiable functions of x in a neighborhood of the point $(3, 1, 2)$.

(b) Let $y(x)$, $z(x)$ be the functions found in part (a). Compute the derivatives $y'(3)$ y $z'(3)$.

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(3) Consider the function

$$f(x, y) = x^2 - y^2 - 2xy - x^3$$

(a) Determine the greatest open and convex set S of $\mathbb{R}^2$ where the function f is concave.

(b) Determine whether f achieves global extrema over the set S that you identified in the answer to the previous question.

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(4) Consider the function

$$f(x, y) = x^2 + y^2 - 2xy$$

and the set

$$A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 2\}$$

(a) Write the Lagrange's equations that are needed to identify the extrema of f on A.

(b) Determine the global extrema of f on A, and clearly argue whether they are a minimum or a maximum.

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(5) Consider the following maximization problem

$$\max_{x,y} \quad x^2 + y^2 + y - 1$$

$$\textbf{s.a.} \quad x^2 + y^2 \le 1$$

(a) Write the Kuhn-Tucker's equations that are needed to identify the extrema of f on A.

(b) Determine the feasible points that satisfy the Kuhn-Tucker's equations.
