University Carlos III
Department of Economics
Mathematics II. Final Exam. May 17th 2019

Last Name:		**Name:**
ID number:	**Degree:**	**Group:**

IMPORTANT

- **DURATION OF THE EXAM: 2h**
- Calculators are **NOT** allowed.
- **Scrap paper:** You may use the last two pages of this exam and the space behind this page.
- **Do NOT UNSTAPLE** the exam.
- You must show a valid ID to the professor.

Problem	Points
1	
2	
3	
4	
5	
6	
Total	

(1) Given the following system of linear equations,

$$\begin{cases} 2x - y + z &= 3 \\ x - y + z &= 2 \\ 3x - y - az &= b \end{cases}$$

where $a, b \in \mathbb{R}$ are parameters.

(a) Classify the system according to the values of a and b. **5 points**

(b) Solve the above system for the values $a = -1$, $b = 4$. **3 points**

(2) Consider the set $A = \{(x, y) \in \mathbb{R}^2 : 0 \le x \le 1, x^2 + 1 \le y \le 3x + 1\}$ and the function

$$f(x, y) = \sqrt{\log(x + y)}$$

defined on A.

(a) Sketch the graph of the set A and justify if it is open, closed, bounded, compact or convex. **5 points**

(b) State Weierstrass' Theorem. Determine if it is possible to apply Weierstrass' Theorem to the function f defined on A. Using the level curves, determine (if they exist) the extreme global points of f on the set A. **5 points**

(3) Consider the function $f(x, y) = 2x + y - \ln x - \ln y$.

(a) Determine its domain and the regions of $\mathbb{R}^2$ where the function is concave or convex. **5 points**

(b) Study the existence of global extreme points for the function f in its domain. **5 points**

(4) Consider the set of equations

$$\begin{aligned} x^2 y + z e^y &= -1 \\ x - y + z &= 0 \end{aligned}$$

(a) Prove that the above system of equations determines implicitly two differentiable functions $y(x)$ and $z(x)$ in a neighborhood of the point $(x, y, z) = (1, 0, -1)$. **3 points**

(b) Compute

$$y'(1), z'(1)$$

and the first order Taylor polynomial of $y(x)$ and $z(x)$ at the point $x_0 = 1$. **5 points**

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(5) Consider the function $f(x, y) = 3axy - x^3 - y^3$, where $a \neq 0$ is a parameter.

(a) Determine the critical points of f in the set $\mathbb{R}^2$. **5 points**

(b) Find and classify, according to the values of the parameter a, the critical points of f. **5 points**

(c) Determine the value of the parameter a for which there is a local maximum where the function attains the value 8 and the value of the parameter a for which there is a local minimum where the function attains the value -1. **5 points**

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(6) Consider the functions $f(x, y) = xy$ and $g(x, y) = x^2 + y^2 - 8$. Let $S = \{(x, y) : g(x, y) = 0\}$.

(a) Explain why $f(x, y)$ must have a global maximum on the set S. **2 points**

(b) Using the Lagrangian method, find the global maxima of $f(x, y)$ on the set S. **7 points**