(1) Given the following system of linear equations,

$$\begin{cases} ax + (3a+1)y + (3a+4)z &= 1 + a - b \\ x + 2y + 3z &= 1 \\ -2ax + (2-2a)y + (7-5a)z &= 3 - 2a - ab - 2b \end{cases}$$

where $a, b \in \mathbb{R}$ are parameters.

(a) Classify the system according to the values of a and b. **1 point**

(b) Solve the above system for the values $a = -1$, $b = -3$. **1 point**

(2) Consider the set $A = \{(x,y) \in \mathbb{R}^2 : 0 \leq x,\ 1 - x^2 \leq y,\ x - 1 \leq y\}$ and the function

$$f(x,y) = \frac{-x-2y}{2}$$

defined on A.

(a) Sketch the graph of the set A and justify if it is open, closed, bounded, compact or convex. **1 point**

(b) Determine if it is possible to apply Weierstrass' Theorem to the function f defined on A. Using the level curves, determine (if they exist) the extreme global points of f on the set A. **1 point**

(3) Consider the function $f(x,y) = x^3 + 2x^2 + 2xy - 16x + \frac{y^2}{2} - 2y - 4$.

(a) Determine the largest open subset of $\mathbb{R}^2$ where the function is strictly concave or convex. **1 point**

(b) Determine the critical points of the function f (if they exist) on $\mathbb{R}^2$. Classify the critical points of f on A. Determine if any of those critical points is a global extreme point. Justify your answer. **1 point**

(4) Consider the set of equations

$$\begin{aligned} -u^3 + v^2 + x^2 - y^2 + 4 &= 0 \\ -2u^2 + 3v^4 + 2xy + y^2 + 8 &= 0 \end{aligned}$$

(a) Prove that the above system of equations determines implicitly two differentiable functions $u(x,y)$ and $v(x,y)$ in a neighborhood of the point $(x,y,u,v) = (2,-1,2,1)$. **0,5 points**

(b) Compute

$$\frac{\partial u}{\partial x}(2,-1), \quad \frac{\partial v}{\partial x}(2,-1) \quad \frac{\partial u}{\partial y}(2,-1), \quad \frac{\partial v}{\partial y}(2,-1)$$

1 point

(c) Using the previous part and Taylor's polynomial of order 1 of the function $u(x,y)$, compute approximately the value of $u(1.99, -1.019)$. **0,5 points**

(5) Consider the function $f(x,y,z) = x^2 + y^2 + z^2 - 3x - 4y$ and the sphere of equation $x^2 + y^2 + z^2 = 25$.

(a) Check that the hypotheses of Lagrange's Theorem hold. Write the Lagrange equations for f on the sphere. compute the points that satisfy those equations and the values of the associated Lagrange multipliers. **1 point**

(b) Assuming that the sphere is closed and bounded and using part (a) above, determine the extreme points of the function f on the sphere. Determine which of those points correspond to global maxima or minima. Justify your answer. **1 point**