(1) Consider the following system of linear equations with a parameter $a \in \mathbb{R}$.

$$\begin{cases} ax + y + z &= b \\ ax + ay + z &= a \\ x + ay + az &= 1 \end{cases}$$

Please, answer the following questions.

(a) Classify the system according to the values of a. **1 point**

(b) Solve the above system for the values $a = b = -1$. **1 point**

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(2) Consider the function $f(x, y) = 4x - y$ and the set

$$A = \{(x, y) \in \mathbb{R}^2 \, : \, 0 \leq x \leq 3, \; 0 \leq y < 9, \; x^2 \leq y\}.$$

(a) Represent the set A, its boundary, closure and interior. Argue whether the function f and the et A satisfy the conditions of Weierstrass' Theorem. **1 point**

(b) Represent the level curves of the function f on the set A, indicating the directions in which f increases/decreases. Using the level curves, determine (if they exist) the global extreme points of f on A. **1 point**

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(3) Consider the function $f(x, y) = bx^2 + y^3 - 6bxy$ with $b \in \mathbb{R}$, $b \neq 0$.

(a) Determine the critical points (if they exist) of the function f on the set $\mathbb{R}^2$. **1 point**

(b) Classify the critical points found above into (local or global) maximum, minimum and saddle points. **1 point**

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(4) Consider the equation $3xz - 8y^3 - z^3 + 6z = 3$.

(a) Prove that the above equation defines a differentiable function $z(x, y)$ in a neighbourhood of the point $(2, 1, 1)$. **1 point**

(b) Compute Taylor's polinomial of order 1 of the function $z(x, y)$, computed above, at the point $(2, 1)$. **1 point**

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(5) Consider the function $f(x, y, z) = 2x + y^2 + z^2$ on the set $A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 9, z = 0\}$

(a) Write the Lagrange equations for f on the set A. Compute the points that satisfy those equations and the value of the corresponding Lagrange multipliers. **1 point**

(b) Knowing that the set A is closed and bounded, study the existence of global extreme points of f on A and compute those points. **1 point**