University Carlos III
Department of Economics
Mathematics II. Final Exam. May 2016

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(1) Consider the following system of linear equations with parameters $a, b \in \mathbb{R}$

$$\left\{ \begin{array}{lcc} ax + y + z & = & b \\ x + ay + z & = & 1 \\ x + (2a-1)y + z & = & 2-b \end{array} \right.$$

(a) Classify the system according to the values of a and b. **1 point**

(b) Solve the above system of linear equations for the values of the parameters $a = b = 1$. **1 point**

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(2) Consider the function $f(x, y) = x - y$ and the set $A = \{(x, y) \in \mathbb{R}^2 : -1 \leq y \leq \sqrt{x}, 0 \leq x \leq 4\}$.

(a) Represent graphically the set A, its boundary, closure and interior. Justify if the function f attains a global maximum and/or a minimum in the set A, without computing them. **1 point**

(b) Using the level curves of the function f, compute the global extreme points of the previous part. **1 point**

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(3) Consider the function

$$f(x, y) = \left\{ \begin{array}{cl} \frac{xy}{x^2+y^2} e^{xy} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{array} \right.$$

(a) Compute the partial derivatives of f at the point $(0, 0)$. **1 point**

(b) Determine if f is continuous and/or differentiable at the point $(0, 0)$. **1 point**

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(4) An agent has a utility function $f(x, y, z) = \ln x + 2\ln y + 3\ln z$. The agent chooses the bundle $(x, y, z) \in \mathbb{R}^3$ which maximizes his utility function $f(x, y, z)$ on his budget line

$$A = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + 3z = 90, x > 0, y > 0, z > 0\}$$

(a) Write the Lagrange equations satisfied by the extreme points of f on the set A. Compute the points which satisfy the Lagrange equations and the value of the Lagrange multiplier, corresponding to each of those points. **1 point**

(b) Classify the solutions of the previous part into local maxima and minima, using the second order conditions. Can we say if any of the previous local maxima and minima is a global maximum or minimum in the set A? Justify the answers. **1 point**

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(5) Consider the function $f(x, y) = 10 - 8x + 2x^2 + 8y - 4xy + bx^2y + 2y^2$ con $b > 2$.

(a) Determine the critical points of the function f on the set $\mathbb{R}^2$. **1 point**

(b) Classify the critical points of f into (local or global) maxima, minima and saddle points, depending on the values of the parameter b. **1 point**
