(1) Given the following system of linear equations, which depends on a parameter $a \in \mathbb{R}$,

$$\left\{\begin{array}{lcr} x + 2y - 3z & = & 4 \\ 3x - y + 5z & = & 2 \\ 4x + y + (a^2 - 14)z & = & a + 2 \end{array}\right.$$

(a) Classify the system of equations depending on the values of the parameter $a$.
(b) Solve the system for the values of the parameter $a$ for which there are infinitely many solutions.

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(2) Consider the set $A = \{(x, y) \in \mathbb{R}^2 : y \leq x + 1, y \leq -x + 1, y > 0\}$ and the function $f(x, y) = y - x^2$.
(a) Draw the set $A$, its boundary and its interior. Determine, justifying the answers, if the set $A$ is closed, open, bounded, compact and/or convex.
(b) Do the hypotheses of Weierstrass' Theorem hold? Why?
(c) Draw the level curves of $f$. Indicate the direction of growth. Using its level curves, determine if the function $f$ attains a global maximum (and/or a minimum) value in the set $A$. Find points at which it is attained.

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(3) Consider a firm that needs to produce 42 units of a certain product at the minimum possible cost. If the firm uses $K$ units of capital and $L$ units of labour, its production is $\sqrt{K} + \sqrt{L}$ units. The prices per unit of capital and labour are, respectively, 1 and 20 monetary units.
(a) Write the optimization problem of the firm. Write the associated Lagrangian funtion and the Lagrange equations.
(b) Solve the Lagrange equations. Check the second order conditions for the critical points and find the solution of the problem.
Suppose now that the firms needs to produce 41 units. Using the computations you have already done and without solving the problem again, determine approximately how many monetary units would the company save in this case.

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(4) Given the following system of equations

$$\left.\begin{array}{rcc} x^2y + y^2z + z^2x & = & 5 \\ x + y + z & = & 2 \end{array}\right\}$$

(a) Apply the Implicit Function Theorem (checking that the hypotheses are satisfied) to show that, in the above system of equations, it is possible to obtain the variables $y$ and $z$ as functions of $x$ in a neighborhood of the point $(x_0, y_0, z_0) = (1, -1, 2)$, so that $x$ and the functions $y(x)$ and $z(x)$ are solutions of the above system of equations that satisfy $y(1) = -1$, $z(1) = 2$.
(b) Compute Taylor's first order approximation of the functions $y(x)$, $z(x)$ around the point $x_0 = 1$.

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(5) Consider the function $f(x, y) = xye^{x+2y}$.
(a) Determine the critical points (if any) of the function $f$ in the set $\mathbb{R}^2$.
(b) Classify the critical points of $f$ into (local or global) maxima, minima and saddle points

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(6) Consider the function $f(x, y) = 2ax^2 - by^2 + 4x - 3$. Discuss, depending on the values of the parameters $a$ and $b$, when the function $f$ is strictly convex and/or concave.

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