University Carlos III
Department of Economics
Mathematics II. Final Exam. May 2012

Last Name:		**Name:**
ID number:	**Degree:**	**Group:**

IMPORTANT

- **DURATION OF THE EXAM: 2h**
- Calculators are **NOT** allowed.
- **Scrap paper:** You may use the last two pages of this exam and the space behind this page.
- **Do NOT UNSTAPLE** the exam.
- You must show a valid ID to the professor.
- Read the exam carefully. Each part of the exam counts 1 point. Please, check that there are 10 pages in this booklet

Problem	Points
1	
2	
3	
4	
5	
Total	

(1) Consider the following system of linear equations

$$\begin{cases} ax + y &= 3 \\ x - az &= 2 \\ y + z &= b \end{cases}$$

where $a, b \in \mathbb{R}$.

(a) Classify the system according to the values of a and b.

(b) Solve the above system for the values of a and b for which the system has infinitely many solutions.

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(2) Consider the set $A = \{(x,y) \in \mathbb{R}^2 : y \geq -x^2 + 1, y \leq -x^2 + 4, x > 0, y \geq 0\}$ and the function

$$f(x,y) = x + \frac{y}{2}.$$

(a) Draw the set A, its boundary and its interior. Determine, justifying your answers, whether the set A is closed, open, bounded, compact and/or convex.

(b) Are the hypotheses of Weierstrass' Theorem satisfied for the set A and the function f? Draw the level curves of f indicating the direction of growth. Use the level curves to determine (if they exist) the global maximum and/or minimum of f on A and the points at which they are attained.

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(3) Answer the following questions.

(a) Given the function $f(x,y) = y \ln xy - 3$, compute the plane tangent to the graph of f corresponding to the point $(x,y) = (1/2, 2)$. Compute the derivative of f at the point $(1/2, 2)$ according to the vector $v = (-1, 3)$

(b) Given the function f above, compute the Taylor polynomial of f of order two around the point $(1/2, 2)$.

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(4) Consider the function $f(x,y) = 8ax^3 - 24xy + y^3$, where $a \neq 0$.

(a) Find the critical points of the function f above.

(b) Classify the critical points found above, according to the values of a.

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(5) Consider the function

$$f(x,y) = x^4 - y^4$$

and the set $A = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$.

(a) Find the Lagrange equations that determine the extreme points of f in A and calculate the solutions of these equations.

(b) Characterize the above solutions into local maxima and minima, using the second order conditions. Can you tell if they are global maxima and/or minima? (Explain your answer)
