(1) Consider the following system of linear equations

$$\begin{cases} 3x + 2y + 2az &= -13 \\ x + 2y + az &= -5 \\ 5x + 2ay - 3z &= -18 \end{cases}$$

where $a \in \mathbb{R}$ is a constant.

(a) Classify the system according to the values of a.

(b) Solve the above linear system for the values of a for which the system has infinitely many solutions. How many parameters are needed to describe the solution?

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(2) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$.

(a) Write the definition that f is continuous at the point $p = (x_0, y_0)$.

(b) Consider the function

$$f(x, y) = \begin{cases} \frac{2x^2y}{x^4+y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

Determine if the function f is continuous at the point $(0, 0)$.

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(3) Consider the function

$$f(x, y) = 2xe^{y/x},$$

the point $p = (2, 0)$ and the vector $v = (3, 2)$.

(a) Compute the gradient of the function f at the point p. Compute the tangent plane to the graph of f at the point $(2, 0, 4)$.

(b) Compute the directional derivative of the function f at the point p in the direction of the vector v. Which is the direction of maximal growth of f at the point p? Which is the maximum value of the directional derivative?

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(4) Consider the function

$$f(x, y, z) = x^2 - y^2 + 2xy - \frac{z^2}{2} + 6$$

and the set

$$A = \{(x, y, z) \in \mathbb{R}^3 : z = 2x + 2y - 1\}$$

(a) Compute the Lagrange equations that determine the extreme values of f on A.

(b) Determine the points that satisfy the Lagrange equations.

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(5) Consider the following maximization problem

$$\max_{x,y} \quad y(x-1)$$

$$\textbf{s.t.} \quad (x-1)^2 + y^2 \leq 2$$

(a) Compute the Kuhn-Tucker equations that determine the extreme values of f on A.

(b) Determine the points that satisfy the Kuhn-Tucker equations.
