(1) Consider the set $A = \{(x,y) \in \mathbb{R}^2 : |y| \leq x^2, \quad x \geq 0\}$.
  (a) Draw the set A, its boundary and interior and determine wether A is open, closed, bounded, compact and/or convex. You must argue your answers.
  (b) Consider the function $f(x,y) = x + y^2$. Determine if this function attains a maximum on A. Does it attain a minimum? If the answer to any of these questions is yes, find the extreme points of f on A.

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(2) Consider the following system of equations

$$\left\{ \begin{array}{rcl} z^3 + 3x - y - t^2 & = & 3 \\ \ln y + x - z^2 & = & 0 \end{array} \right.$$

  (a) Prove that the above system of equations determines the variables y, z as differentiable functions of x and t in a neighbourhood of the point $(x,y,z,t) = (1,1,1,0)$.
  (b) Let $y(x,t)$ be the function whose existence has been proved in part (a). Write the equation of the plane tangent to the graph of y at the point $(x,t;y(x,t)) = (1,0;1)$.

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(3) Suppose a monopoly produces three different products, whose inverse demands are given by the following functions

$$\begin{aligned} p_1 &= 45 - 4q_1 \\ p_2 &= 29 - 3q_2 \\ p_3 &= 21 - 2q_3 \end{aligned}$$

where q_1, q_2 y q_3 are the quantities demanded and p_1, p_2 y p_3 are their respective prices. The cost function is $C(Q) = 20 + 5Q + Q^2$ with $Q = q_1 + q_2 + q_3$.
  (a) Write the function of net profit $B(q_1, q_2, q_3)$ (that is, revenue minus cost) as a function of the amounts demanded. Determine wether the profit function $B(q_1, q_2, q_e)$ is concave or convex.
  (b) Compute the levels of demand q_1, q_2 y q_3 that maximize the profit of the monopoly. Justify that these amounts that you have obtained are a global maximum of the function $B(q_1, q_2, q_e)$.

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(4) Consider the function $f(x,y,z) = 3ax^2 + axz + aby^2 + 3az^2$ with $a, b \in \mathbb{R}$.
  (a) Find the values of $a, b \in \mathbb{R}$ for which the function f is convex.
  (b) Find the values of $a, b \in \mathbb{R}$ for which the function f is concave.

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(5) Consider the following maximization problem $f(x,y) = x^2 - 6x + y^2 + 9$ in the set $A = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 4\}$.
  (a) Justify that the function f has extreme points (i.e. maxima and/or minima) on the set A. Write the Kuhn-Tucker equations that determine the extreme points of f on A.
  (b) Compute the points that satisfy the Kuhn-Tucker equations. Determine at which points does the function f attain a maximum on A and at which points does it attain a minimum.
