University Carlos III
Department of Economics
Mathematics II. Final Exam. June 2009

Last Name:		**Name:**
ID number:	**Degree:**	**Group:**

IMPORTANT

- **DURATION OF THE EXAM: 2h**
- Calculators are **NOT** allowed.
- **Scrap paper:** You may use the last two pages of this exam and the space behind this page.
- **Do NOT UNSTAPLE** the exam.
- You must show a valid ID to the professor.
- Read the exam carefully. Each part of the exam counts 1 point. Please, check that there are 10 pages in this booklet

Problem	Points
1	
2	
3	
4	
5	
Total	

(1) Consider the following system of linear equations

$$\begin{cases} x + (k+1)y + 2z & = & -1 \\ kx + y + z & = & k \\ (k-1)\,x - 2y - z & = & k+1 \end{cases}$$

where $k \in \mathbb{R}$ is a constant.

(a) Classify the system according to the values of k.

(b) Solve the above system for the values of k for which the the system has infinitely many solutions. How many parameters are needed to describe the solution?

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(2) Consider the function $f(x,y) = 3x\ln(x^2 - y)$, the point $p = (2,3)$ and the vector $v = (1,2)$.

(a) Find the directional derivative of the function f at the point p in the direction of the vector v.

(b) What is the direction of maximal growth of f at the point p? What is the maximal value of the directional derivative of f at the point p?

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(3) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x,y) = \begin{cases} \frac{5xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

(a) Study if the function f is continuous at the point $(0,0)$. Study at which points of $\mathbb{R}^2$ the function f is continuous.

(b) Argue in which of the following sets we may apply the Theorem of Weierstrass to show that the above function f attains a global extremum on the set

$$A = \{(x,y) \in \mathbb{R}^2 : (x-1)^2 + y^2 \leq 4\}$$
$$B = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \geq 1\}$$
$$C = \{(x,y) \in \mathbb{R}^2 : (x-3)^2 + y^2 \leq 1\}$$

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(4) A company sells two goods A and B. The total profits obtained from the sale of x_1 units of A and x_2 units of B is the following:

$$I(x_1, x_2) = -x_1^2 - 3x_2^2 - 2x_1x_2 + 4x_1 + 8x_2 \qquad (x_1 \text{ and } x_2 \text{ in thousands of units})$$

(a) Find the amounts x_1 and x_2 which maximize the profits.

(b) Argue why can we be sure that the amount obtained in part (a) is a global maximum of the function $I(x_1, x_2)$

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(5) Consider the following maximization problem

$$\begin{array}{rc} \max\limits_{x,y} & (x-1)^2 - y \\ \textbf{s.t.} & -2x + y \leq 2 \\ & x + y \leq 5 \\ & y \geq 0 \end{array}$$

(a) Find the Kuhn-Tucker equations that determine the extreme points of f and draw the feasible set.

(b) Which restrictions are binding at the point $A = (1,0)$? Show that the point $A = (1,0)$ verifies the Khun-Tucker equations for the problem above.
