University Carlos III
Department of Economics
Mathematics II. Final Exam. June 2016

---

(1) Given the following system of linear equations with a parameter $a \in \mathbb{R}$

$$\begin{cases} ax+y+z &= 1 \\ x+ay+z &= a \\ x+y+az &= a^2 \end{cases}$$

(a) Classify the system according to the values of a. **1 point**

(b) Solve the above system for the value of the parameter $a = 1$. **1 point**

---

(2)

(a) Show that the following system of equations

$$\begin{aligned} y^2+z^2-x^2+2 &= 0 \\ yz+xz-xy-1 &= 0 \end{aligned}$$

defines two functions $y = y(x)$, $z = z(x)$ in a neighborhood of the point $(x, y, z) = (2, 1, 1)$. Compute $y'(2)$, $z'(2)$. **1 point**

(b) Consider the functions $F(x, y, z) = xz - y^2$ and $G(x) = F(x, y(x), z(x))$. Compute $G'(2)$. **1 point**

---

(3) Consider the function $f(x, y) = \frac{x^2}{y} + ay^2$ with $a \in \mathbb{R}$ defined on the open set $D = \{(x, y) \in \mathbb{R}^2 : y > 0\}$.

(a) Study the convexity of the function f in the set A, depending on the values of the parameter a. **1 point**

(b) Compute the Taylor polynomial of degree 2 of the function f at the point $p = (0, 1)$. **1 point**

---

(4) Consider the function $f(x, y, z) = x - y + z$ and the set $A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 9, x + z = 4\}$.

(a) Compute the Lagrange equations that determine the extreme points of the function f in the set A. Compute the points that satisfy the Lagrange equations and the values of the corresponding Lagrange multipliers at each of the points. **1 point**

(b) Using the second order conditions, classify the solutions found in the previous part into maxima, minima and local points. Can you say if any of the local maxima and/or maxima is a global extreme point on the set A? Justify adequately your answers. **1 point**

---

(5) Consider the function $f(x, y) = x^4 + y^3 - 2a^2x^2 - 3y$ with $a \in \mathbb{R}$, $a \neq 0$.

(a) Determine the critical points of the function f in the set $\mathbb{R}^2$. **1 point**

(b) Classify the critical points of the previous part into (local and/or global) maxima and saddle points. **1 point**