University Carlos III
Department of Economics
Mathematics II. Final Exam. May 31st 2021

Last Name:		**Name:**
ID number:	**Degree:**	**Group:**

IMPORTANT

- **DURATION OF THE EXAM: 2h**
- Calculators are **NOT** allowed.
- **Scrap paper:** You may use the last two pages of this exam and the space behind this page.
- **Do NOT UNSTAPLE** the exam.
- You must show a valid ID to the professor.

Problem	Points
1	
2	
3	
4	
5	
Total	

(1) Given the following system of linear equations,

$$\begin{cases} ax + y + z &= b \\ ax + y + az &= a \\ x + ay + z &= 1 \end{cases}$$

where $a, b \in \mathbb{R}$ are parameters.

(a) Classify the system according to the values of a and b. **10 points**

(b) Solve the above system when $a = -1$. **10 points**

(2) Consider the set

$$A = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 4, \;\; y \leq 2 - x, \;\; y \geq x - 2\}$$

and the function

$$f(x,y) = -\ln\left((x+1)^2 + y^2\right)$$

(a) Sketch the graph of the set A, its boundary and its interior and justify if it is open, closed, bounded, compact or convex. **10 points**

(b) State Weierstrass' Theorem. Determine if it is possible to apply Weierstrass' Theorem to the function f defined on A. Using the level curves, determine (if they exist) the extreme global points of f on the set A. **10 points**

(3) Consider the function $f(x,y) = x^2 + 4xy - 2x + 2y^3 + 6y^2 - 20y$.

(a) Determine the largest open subset of $\mathbb{R}^2$ where the function f is strictly concave or convex. **10 points**

(b) Find the critical points of f. Classify the critical points into (local/global) maxima/minima or saddle points. **10 points**

(4) Consider the set of equations

$$\begin{aligned} xz + yz &= -4 \\ x + y^2 - z &= -2 \end{aligned}$$

(a) Prove that the above system of equations determines implicitly two differentiable functions $y(x)$ and $z(x)$ in a neighborhood of the point $(x,y,z) = (-1,-1,2)$. **10 points**

(b) Compute

$$y'(-1), z'(-1)$$

and the first order Taylor polynomial of $y(x)$ and $z(x)$ at the point $x_0 = -1$. **10 points**

(5) Consider the extreme points of the function

$$f(x,y) = 2x^3 - y^3$$

in the set

$$S = \{(x,y) : x^2 + y^2 = 5\}$$

(a) Write the Lagrangian function and the Lagrange equations and their solutions. **10 points**

(b) Find the global maximum and minimum values of f on the set S. **10 points**