University Carlos III of Madrid

Department of Economics
Mathematics II. Final Exam. September 2005.

IMPORTANT:

- **DURATION OF THE EXAM: 2h. 30min.**
- Calculators are **NOT** allowed.
- **Hand in this booklet.** Do not hand in scratch paper. Only the answers written on this booklet will be graded.
- You must show a valid ID to the professor.
- Each part of the exam counts $0'5$ points.

Last Name: **Name:**

DNI: **Group:**

(1) Consider the following system of linear equations

$$\begin{aligned} x + by + z &= 1 \\ y + az &= b \\ x + (b-1)y + 2z &= 1 \end{aligned}$$

where a and b are parameters.

(a) Determine, according to the values of a and b, whether the system is compatible, incompatible, determinate or indeterminate.

(b) Solve the system for the values of a and b for which the system is compatible indeterminate.

(2) Given the matrix

$$A = \begin{pmatrix} 2 & a & a+1 \\ 0 & 2 & 0 \\ 0 & a & 1 \end{pmatrix}$$

(a) Compute the characteristic polynomial and the eigenvalues.

(b) For the value $a = 0$, check that the matrix A is diagonalizable and find the matrix change of basis.

(c) Study for which values of $a \neq 0$, the matrix A is diagonalizable.

(3) Given the vectors $\{(-1,2,1,-3),(3,-1,1,-2),(2,1,2,-5),(2,-4,-2,6)\}$ in $\mathbb{R}^4$.

(a) Compute the dimension of the linear subspace generated by them.

(b) Obtain the linear equations that determine the linear subspace generated by them.

(4) Let $A = \{(x,y) \in \mathbb{R}^2 : x \geq 0, x + |y| \leq 1\}$.

(a) Draw the set A.

(b) Draw the boundary, the interior and the closure of the set A. Is A closed, open, convex, bounded or compact?

(c) Consider the function

$$f(x,y) = \begin{cases} \frac{x^2+y^2}{(1-x)^2} & \text{If } x \neq 1, \\ 0 & \text{If } x = 1. \end{cases}$$

Determine if the function f attains a maximum or a minimum in the set A.

(5) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x,y) = \begin{cases} \frac{y^2-x^3y}{x^2+y^2} & \text{if } (x,y) = (0,0), \\ 0 & \text{if } (x,y) \neq (0,0). \end{cases}$$

(a) Study if the function f is continuous at the point $(0,0)$).

(b) Determine whether the partial derivatives of f at the point $(0,0)$) exist.

(c) Study if the function f is differentiable at the point $(0,0)$).

(6) Consider the function $f(x,y) = x^4 + x^2y^2 + y^4 - 3x - 8y$,

(a) Compute the Taylor polynomial of order two of f around the point $(0,0)$.

(b) Determine if the function f is concave or convex in $\mathbb{R}^2$.

(7) Given the function $f(x,y) = y^2x - ay^2 - ax^2$

(a) Show that $(0,0)$ is a critical point of f for any value of a and study its type of critical, according to the values of the parameter a.
(b) For $a=1$ find all the critical points of f.
(c) For $a=1$ classify the critical points of f obtained part b).

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(8) Consider the function $f(x,y,z)=x+y+z$ defined on the set $A=\{(x,y,z)\in\mathbb{R}^3 : x^2+2y^2+4z^2=1\}$.
(a) Find the Lagrange equations that determine the extreme points of f on A.
(b) Determine the points that satisfy the Lagrange equations and classify all the extreme points of f on A.
