(1) Consider the following system of linear equations, with two parameters $a, b \in \mathbb{R}$

$$\left\{\begin{array}{lcl} ax + 2z & = & 2 \\ 5x + 2y & = & 1 \\ x - 2y + bz & = & 3 \end{array}\right.$$

(a) Classify the above linear system of linear equations according to the values of the parameters a and b.

(b) Solve the above linear system of linear equations for the values of the parameters a and b for which there are infinitely many solutions.

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(2) Consider the function

$$f(x,y) = \left\{\begin{array}{ll} \frac{xy}{\sqrt{x^2+y^2}}, & \text{if } (x,y) \neq (0,0) \\ 5, & \text{if } (x,y) = (0,0) \end{array}\right.$$

(a) Prove that the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exists.

(b) Determine the points where the function f is continuous. It is necessary to justify the answer.

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(3) Consider the function $z = e^{3x-2y}$.

(a) Study the subset of $\mathbb{R}^2$ where the function z is convex.

(b) Compute Taylor's polynomial of order 2 of the function z at the point $p = (0,0)$.

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(4) Consider the function $f(x,y) = x^2 + 2y^2$ and the set $A = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$.

(a) Write the Lagrange equations that determine the extreme points of f in the set A and find the solutions of the Lagrange equations.

(b) Using the second order conditions, classify the solutions obtained in part a) into local maxima and minima. Can you tell if one them is a global maximum or a minimum? Please, justify your answer.

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(5) Consider the function $f(x,y) = 10 - 8x + 2x^2 + 8y - 4xy + x^2y + 2y^2$

(a) Determine the critical points (if there are any) of the function f in the set $\mathbb{R}^2$.

(b) Classify the critical points found in the previous part into (local or global) maxima, minima and saddle points.
