**University Carlos III**
**Department of Economics**
**Mathematics II. Final Exam. June 2012**

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|---|---|---|
| **Last Name:** | | **Name:** |
| **ID number:** | **Degree:** | **Group:** |

**IMPORTANT**

- **DURATION OF THE EXAM: 2h**
- Calculators are **NOT** allowed.
- **Scrap paper:** You may use the last two pages of this exam and the space behind this page.
- **Do NOT UNSTAPLE** the exam.
- You must show a valid ID to the professor.
- Read the exam carefully. Each part of the exam counts 1 point. Please, check that there are 10 pages in this booklet

| Problem | Points |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Total | |

(1) Consider the following system of linear equations

$$\begin{cases} x - by - 2z &= 1 \\ x - az &= b \\ x + (2-b)y &= 1 \end{cases}$$

where $a, b \in \mathbb{R}$.

(a) Classify the system according to the values of $a$ and $b$.

(b) Solve the above system for the values of $a$ and $b$ for which the system has infinitely many solutions.

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(2) Consider the function

$$f(x,y) = \begin{cases} \frac{x^2\sqrt{|y|}}{x^2+y^2} & \text{si } (x,y) \neq (0,0), \\ 0 & \text{si } (x,y) = (0,0). \end{cases}$$

(a) Determine whether the function $f$ is continuous at the point $(0,0)$.

(b) Compute (if they exist) the partial derivatives of $f$ at the point $(0,0)$. Compute (if it exists) the derivative of $f$ at the point $(0,0)$ according to the vector $v = (1,4)$. ¿Is the function $f$ differentiable at the point $(0,0)$?

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(3) Consider the function $f(x,y) = ax^2 + (a+b)y^2 + 2axy + 2$, with $a, b \in \mathbb{R}$

(a) Study the concavity and the convexity of the function $f$ according to the values of $a$ and $b$.

(b) For the values $a = 1$, $b = 0$, does the function $f$ attain a maximum and/or minimum value in $\mathbb{R}^2$? At what points? (Justify the answer).

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(4) Consider the function

$$f(x,y) = xy^2$$

and the set $A = \{(x,y) \in \mathbb{R}^2 : x + y \leq 100, 2x + y \leq 120\}$.

(a) Compute the Kuhn-Tucker equations that determine the extreme points of $f$ in $A$.

(b) Compute the solutions of the above Kuhn-Tucker equations.

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(5) Consider the function

$$f(x,y) = x^2 + y^2$$

and the set $A = \{(x,y) \in \mathbb{R}^2 : x - 2y + 6 = 0\}$.

(a) Compute the Lagrange equations that determine the extreme points of $f$ in $A$ and find their solutions.

(b) Using the second order conditions, characterize the above solutions of the Lagrange equations as corresponding to maximum or minimum values. Can you justify if any of those points corresponds to a global maximum or minimum? (Justify the answer)

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