WORKSHEET 1: Introduction

1. For each one of the following inequalities, determine the set of real numbers that satisfy them. Draw that set.

   a)(*) $|9-2x|<1$ b)(*)$-5|x+3|<4x-5$ c)(*)$\dfrac{|x|}{3}+2<|x|$ d)(*) $1<|3-2x|$

   e)(*) $\dfrac{(x^2-16)(x-1)}{x-3}\geq 0$ f) $|x-3|+|x+3|<10$ g) $|x-3|+|x+3|<\alpha,\ \alpha\in\mathbb{R}$ h)$|\dfrac{x-1}{x}|-1\geq 0$

2. (*)Interpret geometrically the inequalities a), b), c) and d) using the functions

   a) $y=|9-2x|$; $y=1$ b) $y=-5|x+3|$; $y=4x-5$

   c) $y=\dfrac{|x|}{3}+2$; $y=|x|$ d) $y=1$; $y=|3-2x|$

3. Discuss if the following inequalities are satisfied :

   a) (*) $|x+y|\leq|x|+|y|$ b)(*) $|x|+|y|\leq|x+y|$ g)(*) $|x-y|\leq|x|+|y|$

   c)(*) $|x-y|\leq|x|-|y|$ d)(*) $|x|-|y|\leq|x-y|$ h)(*) $|x|+|y|\leq|x-y|$

   e)(*) $||x|-|y||\leq|x|+|y|$ f) $|x|+|y|\leq||x|-|y||$ i) $||x|-|y||\leq|x|-|y|$

4. Discuss if the following statements are true or false

   a) $x<y\Rightarrow x^2<\ y^2$ b) $|x|<|y|\ \ \Rightarrow x^2<\ y^2$

   c) $x^2<y^2\Rightarrow x<\ y$ d) $x^2<y^2\Rightarrow|x|<|y|$

5. For the sets $A\subset\mathbb{R}$ that are defined below, obtain the maximun and the minumun, if they exist, for $\alpha=-1$, $\alpha=0$ and $\alpha=1$

   a) $A=\{x: senx=\alpha\}$ b) $A=\{x:\cos x=\alpha\}$ c) $A=\{x:e^x\leq\alpha\}$

   d) $A=\{x:e^x\geq\alpha\}$ e) $A=\{x:\ln x\leq\alpha\}$ f) $A=\{x:\ln x\geq\alpha\}$

6. (*)In $\mathbb{R}^2=\mathbb{R}\times\mathbb{R}$ the following relation is defined: $(a,b)\leq(c,d)$ if and only if $a\leq c$ and $b\leq d$. Prove that "$\leq$" is a partial order relation.

   Let $A=\{(x,y)\in\mathbb{R}^2\mid x+y\leq 1\}$, $B=\{(x,y)\in\mathbb{R}^2|\ |x|\leq 1;\ |y|\leq 1\}$, $C=\{(x,y)\in\mathbb{R}^2|\ 0\leq y\leq 4-x^2\}$ $D=\{(x,y)\in\mathbb{R}^2|\ x^2-9\leq y\leq 0\}$ and $E=\left\{(x,y)\in\mathbb{R}^2:|x|\leq y\leq 6-x^2\right\}$.

   Obtain for the previous sets, if they exist, the maximun and the minimun, the maximals and the minimals.

7. (*)Let $f(x)=1/x$ and $g(x)=x^2-1$.

   a) Find the domain and the range of those functions.
   b) Find $f(g(2))$ and $g(f(2))$.
   c) Find $f(g(x))$ and $g(f(x))$.

8. (*)Find the domain and the range of the following functions:

   a)$f(x)=ln(senx)$ b)$g(x)=ln(sen^2x)$

   c) $h(x)=ln\sqrt{-x^2+4x-3}$

9. Review the graph of the functions:

   a)(*) $f(x)=x^2$ b)(*) $f(x)=e^x$ c)(*) $f(x)=lnx$ d) $f(x)=senx$

   In each case draw the graph of the following functions from the previous ones, interpreting geometrically the results.

   i) $g(x)=f(x+1)$ ii) $h(x)=-2f(x)$ iii) $p(x)=f(3x)$

   iv) $s(x)=f(x)+1$ v) $r(x)=|f(x)|$ vi) $m(x)=f(|x|)$

10. (*)Let $f,\ g:I\to\mathbb{R}$ be increasing funcitons. Discuss if the following statements are true or false

   a) $f+g:I\to\mathbb{R}$ is an increasing function
   b) $f\cdot g:I\to\mathbb{R}$ is an increasing function
   c) $f-g:I\to\mathbb{R}$ is an increasing function if both functions are positive
   d) $f-g:I\to\mathbb{R}$ is an increasing function if both functions are negative

11. Let f, $g : \mathbb{R} \to \mathbb{R}$ be monotonic functions. Discuss when will $g \circ f$ be increasing or decreasing depending on the behaviour of f and g (four cases in total).

12. For each one of the following functions, for example f, find the intervals I, J for $f : I \to J$ to be bijective.
    $a)\ f(x) = x^2; b)\ g(x) = \ln|x|\ ; c)\ h(x) = sen(x);\ d)\ i(x) = e^{-x^2}$

13 (*)Calculate the inverse function of the following functions:

$$f(x) = (x^3 - 5)^5, \quad g(x) = (\sqrt[3]{x} - 5)^5 \quad h(x) = \ln(\tfrac{x-1}{x-2}); i(x) = \tfrac{3x-1}{x-3};\ j(x) = \begin{cases} x+3 & -3 \le x \le 0 \\ -2x & 0 < x \le 3 \end{cases}$$

a) $f^{-1}(x) = \sqrt[3]{5 + \sqrt[5]{x}}$.

b) $g^{-1}(x) = 5 + x^{3/5} = 5 + (\sqrt[5]{x})^3$.

c) $h^{-1}(x) = \frac{2e^x - 1}{e^x - 1}$ d) $i^{-1}(x) = i(x)$ e) $j^{-1}(x) = \begin{cases} x-3 & 0 \le x \le 3 \\ -x/2 & -6 \le x < 0 \end{cases}$

14. Determine if the following functions are even, odd or neither of them:

    a) $f(x) = cos5x$ b)$g(x) = sen2x$ c)$h(x) = cos5xsen2x$ d)$k(x) = \dfrac{x^2}{x^2+1}$
    e)$l(x) = \dfrac{x^3}{x^4+1}$ f)$m(x) = \dfrac{x^3}{x^5+1}$ g) $n(x) = \dfrac{arctgx}{x}$

15. Let f be an even function and g an odd function. Prove that:
    $|g|$ is even; $f \circ g$ is even; $g \circ f$ is even;
    $f \cdot g$ is odd; g^k is even (if k is even); g^k is odd (if k is odd)

16. Determine which of the following functions are periodic and calculate its period.
    $f(x) = sen4x \quad g(x) = tg(\dfrac{x}{3}) \quad l(x) = sen(3x + 2)$

17. Let f be any function and g a periodic function. Is it possible to state that $f \circ g$ and $g \circ f$ are periodic?
    Justify that $f(x) = \dfrac{tg^2 3x + ln(tg3x)}{1 + tg3x}$ is perodic.