UC3M
TOPICS Advanced Mathematics for Economics
Final January 17th, 2023

Niu: ____________ Group: ______

Name: ______________________________

Question:	1	2	3	4	5	Total
Points:	20	15	10	15	10	70
Score:						

Instructions:

- **DURATION OF THE EXAM: 2h**.
- Calculators are **NOT** allowed.
- **DO NOT UNSTAPLE** the exam.
- You must show a valid ID to the professor if required.
- Read the exam carefully. The exam has 5 questions, for a total of 70 points.
- Remember that for a complex number $z = a + ib$, the module is $\rho = |z| = \sqrt{a^2 + b^2}$, and the argument θ is the angle such that $\tan\theta = b/a$.

  Table of usual trigonometric values

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	∞

$\boxed{1}$

Consider a market where only one good is traded where the demand at time t is $D_t = 100 - P_t$ and the supply at time y is $S_t = P_{t-2}$, where P_{t-2} and P_t are the prices at times $t-2$ and t, respectively.

(a) (5 points) Find a difference equation, linear and of order 2, satisfied by the equilibrium prices. *Note:* The equilibrium prices satisfy $D_t = S_t$ for all t.

(b) (5 points) Find the general solution of the difference equation of part (a) above.

(c) (5 points) Find the solution of the difference equation of part (a) above which satisfies the initial conditions $P_0 = 40$, $P_1 = 20$.

(d) (5 points) Fin the maximum and the minimum equilibrium prices.

$\boxed{2}$

Consider the following linear system of difference equations

$$\begin{pmatrix} x_{t+1} \\ y_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} -1 & 0 & \frac{3}{2} \\ 1 & \frac{1}{2} & -1 \\ -\frac{1}{2} & 0 & 1 \end{pmatrix} \begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}.$$

(a) (5 points) Find the equilibrium point and classify it (unstable, stable, locally asymptotically stable or globally asymptotically stable).

(b) (5 points) Find the general solution of the system.

(c) (5 points) Find the solution of the system which satisfies the initial conditions $x_0 = 0$, $y_0 = 8$ and $z_0 = 2$.

$\boxed{3}$

Answer the following questions.

(a) (5 points) Find the general solution of the ODE

$$(1+t)x' + x = t(t+1).$$

(b) (5 points) Find the solution of the ODE

$$(1+t)x' + x = t(t+1)$$

which satisfies $x(0) = x(1)$.

$\boxed{4}$

Find the general solution of the ODE

$$x'' - x' = e^{at},$$

where $a \in \mathbb{R}$, in each of the following cases:

(a) (5 points) When $a = 0$.

(b) (5 points) When $a = 1$.

(c) (5 points) When $a \neq 0$ and $a \neq 1$.

$\boxed{5}$

Consider the following system of differential equations

$$\begin{cases} x' &= -2x - y \\ y' &= x - 2y \end{cases}$$

Find and classify the equilibrium point. Skecht the phase diagram.