ADVANCED MATHEMATICS FOR ECONOMICS – 2014/2015

Sheet 5. Differential Equations (1)

5-1. Is the equation $\dot{x}(t) = x(t^2)$ an ordinary differential equation?

5-2. Check that $x(t) = \pm\sqrt{\ln(C(t^2+1))}$, where C is a positive constant, is solution of the ODE

$$\dot{x}(t) = \frac{t}{x(t)(t^2+1)}.$$

5-3. Write the second order ODE

$$\ddot{x}(t) + a(t)\dot{x}(t) + b(t)x(t) = c(t)$$

as a first order system.

5-4. It is snowing with regularity. At 12 am, a snow plow began to remove snow. The machine took 2 km. in the first hour, and only 1 km. in the second hour. Knowing that the snow plow remove a constant amount of snow per unit of time, When did it start snowing?

5-5. Find the solution of the following problems:

(a) $\dot{x} = \dfrac{e^t}{x(1+e^t)}$.

(b) $\dot{x} = e^{t-x}$, $x(0) = 1$.

5-6. Show that a separable equation is exact.

5-7. Check whether the following ODEs are exact and solve them.

(a) $(\alpha t + \beta x)\,dt + (\beta t + \gamma x)\,dx = 0$.

(b) $(-2tx^3 + t\ln t)\,dt - (3t^2x^2)\,dx = 0$.

5-8. Consider the following supply and demand functions: $Q_s(P) = P - 6$, $Q_d(P) = 15 - 2P$. The prices depends on time, $P(t)$, which derivative satisfies

$$\dot{P} = 2(Q_d(P) - Q_s(P)).$$

Calculate $P(t)$, the equilibrium price and the equilibrium quantity and study if the price converges to the equilibrium price in the long run.